Outline

- Introduction
- Features and Feature Matching
- **Geometry of Image Formation**
- Calibration
- Structure from Motion
- Dense Stereo
- Conclusion

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Camera Obscura

Example image of the camera obscura has been removed in this version due to copy right reasons.
Please refer to the link below.

- principle has been known for a long time (e.g. Aristotle)
- box with infinitesimally small hole
- light falls through the hole and image is projected onto wall facing the hole

Image: https://en.wikipedia.org/wiki/Camera_obscura

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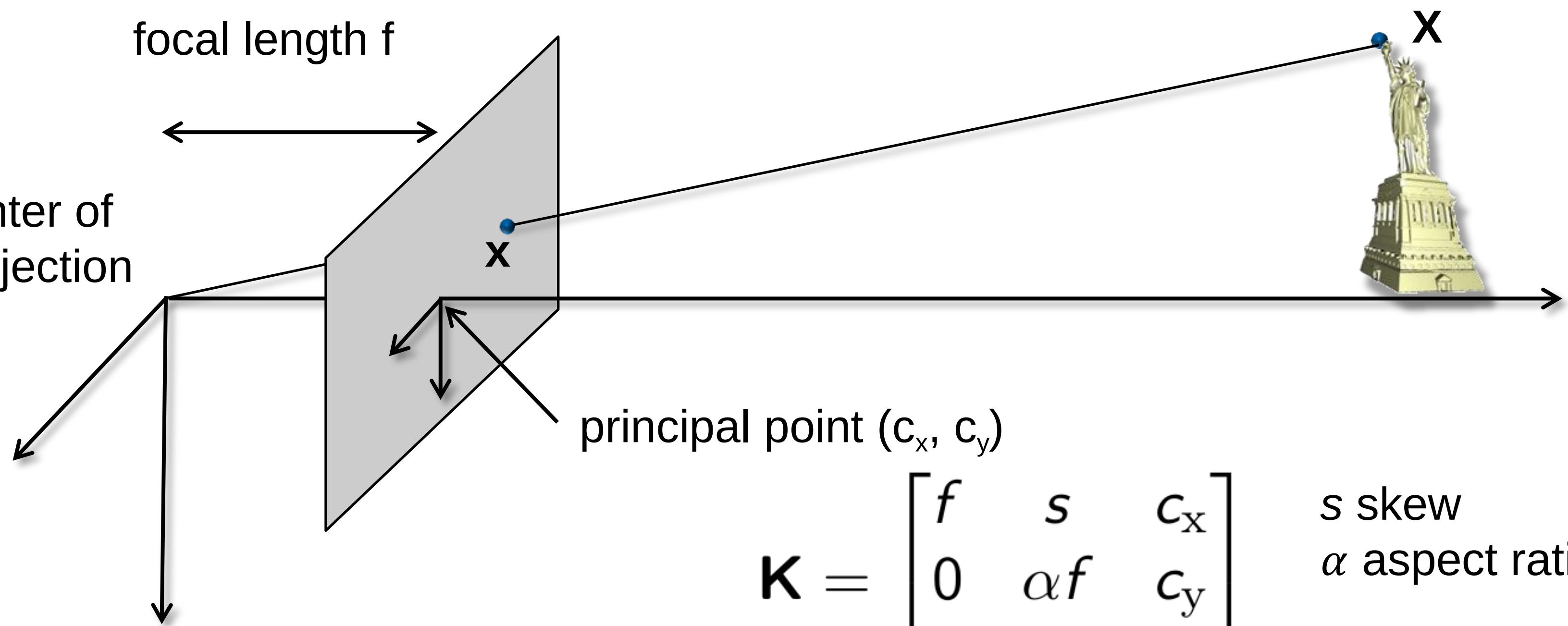
Pinhole Camera Model



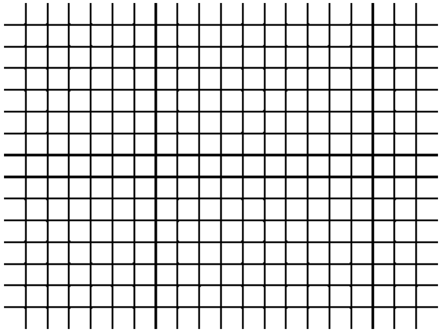
$$\mathbf{K} = \begin{bmatrix} f & s & c_{\mathrm{x}} \\ 0 & \alpha f & c_{\mathrm{y}} \\ 0 & 0 & 1 \end{bmatrix}$$

s skew
α aspect ratio

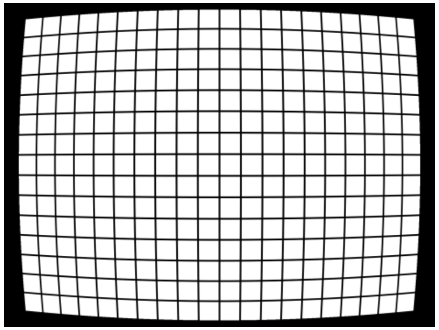
- all rays meet in center of projection (**single-view-point** camera – SVP camera)

Hartley 04

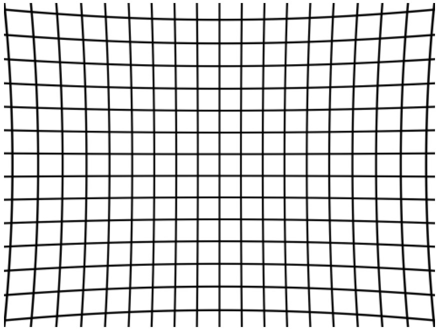
Lens Distortion



distortion-free image



barrel distortion



cushion distortion

- radial lens distortion modeled by polynomial for image coordinates (*x*, *y*):

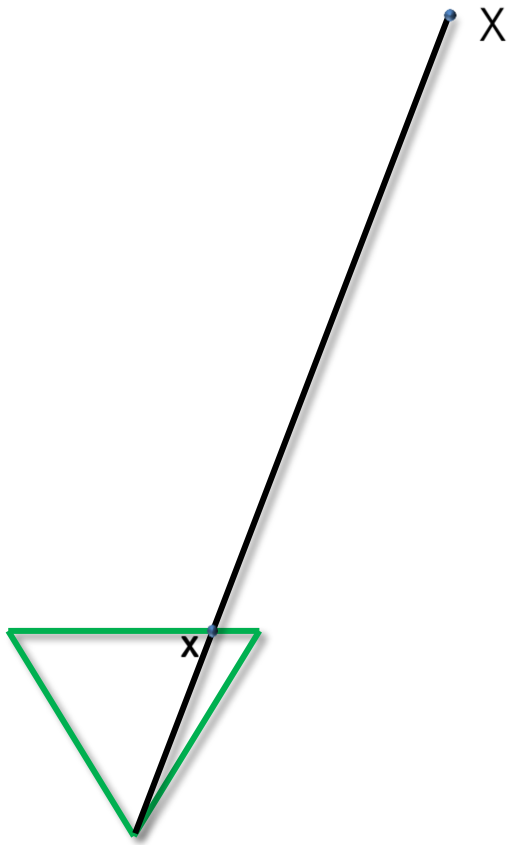
$$r^2 = x^2 + y^2$$

$$x_\mathrm{d} = x(1 + k_1 r^2 + k_2 r^4)$$

$$y_\mathrm{d} = y(1 + k_1 r^2 + k_2 r^4)$$

Szeliski 2011, Mc Glone 2004

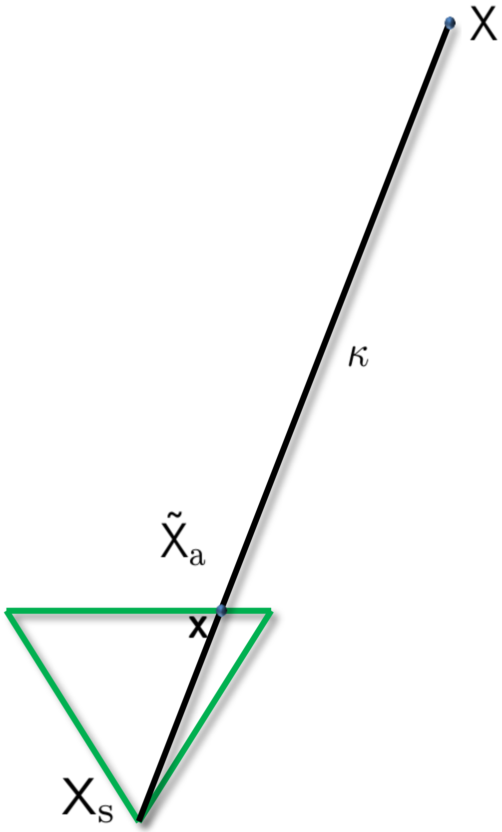
3D-2D Projection



$$\underbrace{\begin{bmatrix} x \\ y \\ w \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} f & s & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\mathbf{X}}$$

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

2D-3D Projection



$$\mathbf{x} = (x, y, 1)^{\mathrm{T}}$$

$$X_s = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \tilde{X}_a = \frac{\mathbf{K}^{-1}\mathbf{x}}{||\mathbf{K}^{-1}\mathbf{x}||_2}$$

$$X = X_s + \kappa \tilde{X}_a$$

Hartley 2004

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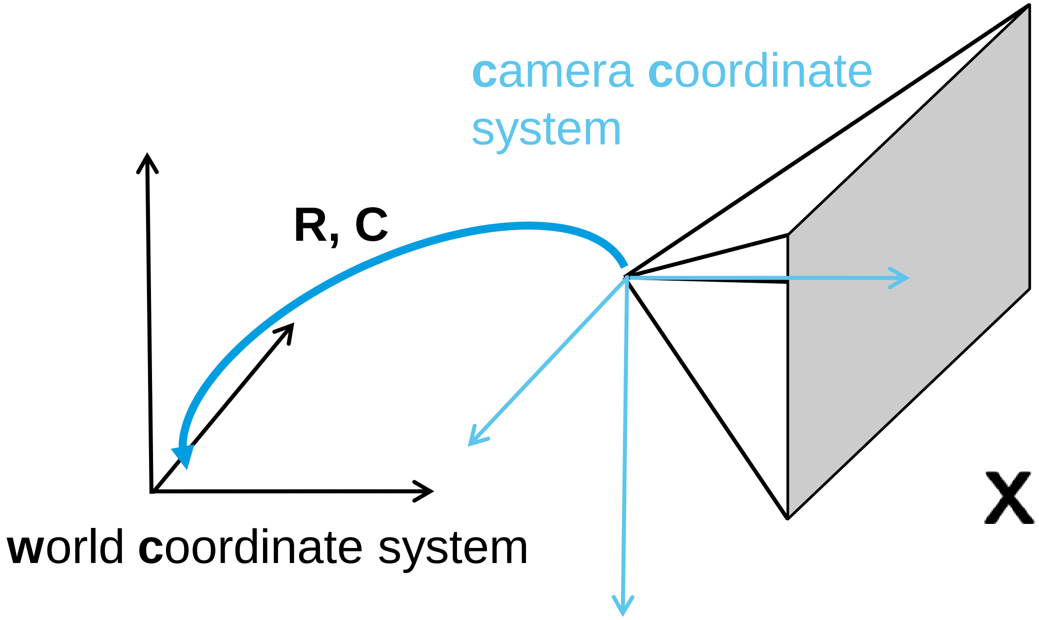
Rotation and Translation in Space



$$X^{cc} = \mathbf{R}^T X^{wc} - \mathbf{R}^T C$$

$$\mathbf{X}^{cc} = \begin{pmatrix} \mathbf{R}^T & -\mathbf{R}^T C \\ 0^T & 1 \end{pmatrix} \mathbf{X}^{wc}$$

$$X^{wc} = \mathbf{R} X^{cc} + C$$

$$X^{wc} = \mathbf{R} X_s + C + \kappa \mathbf{R} \tilde{X}_a$$

Hartley 04

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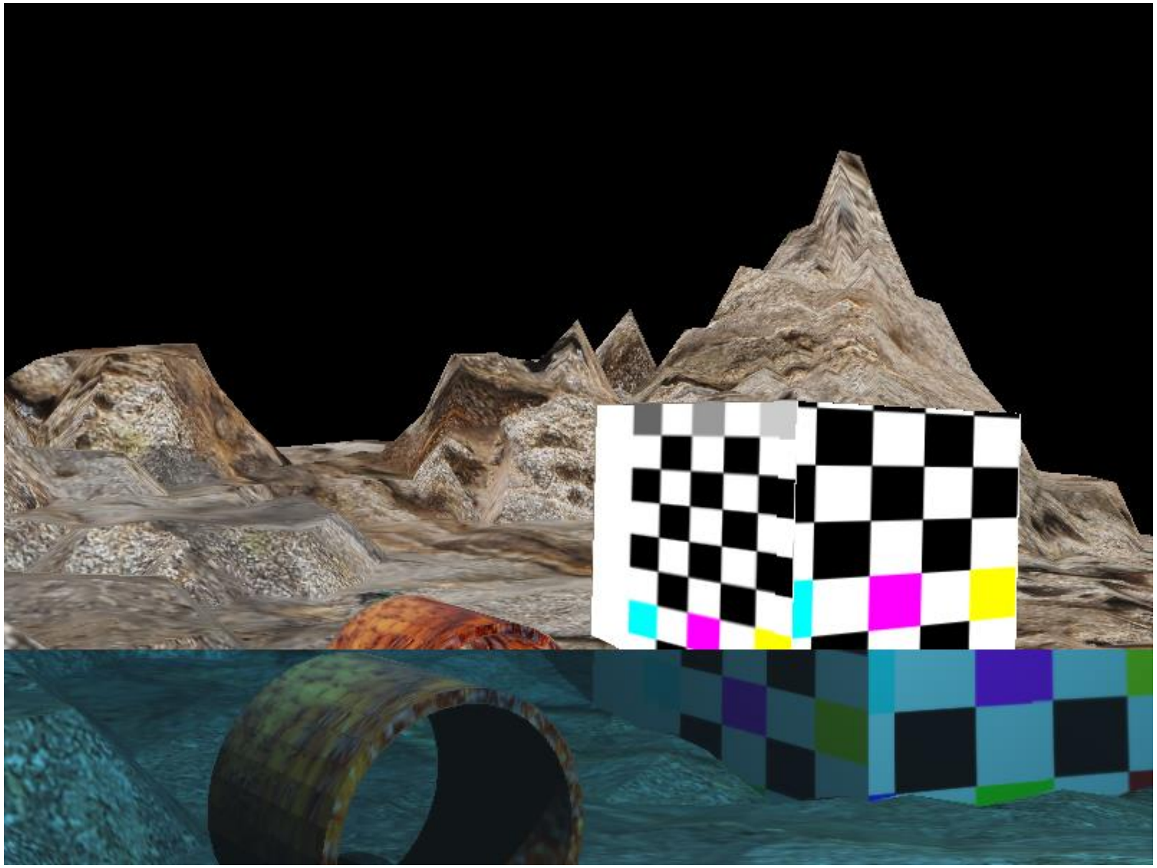
Refraction at Underwater Housings

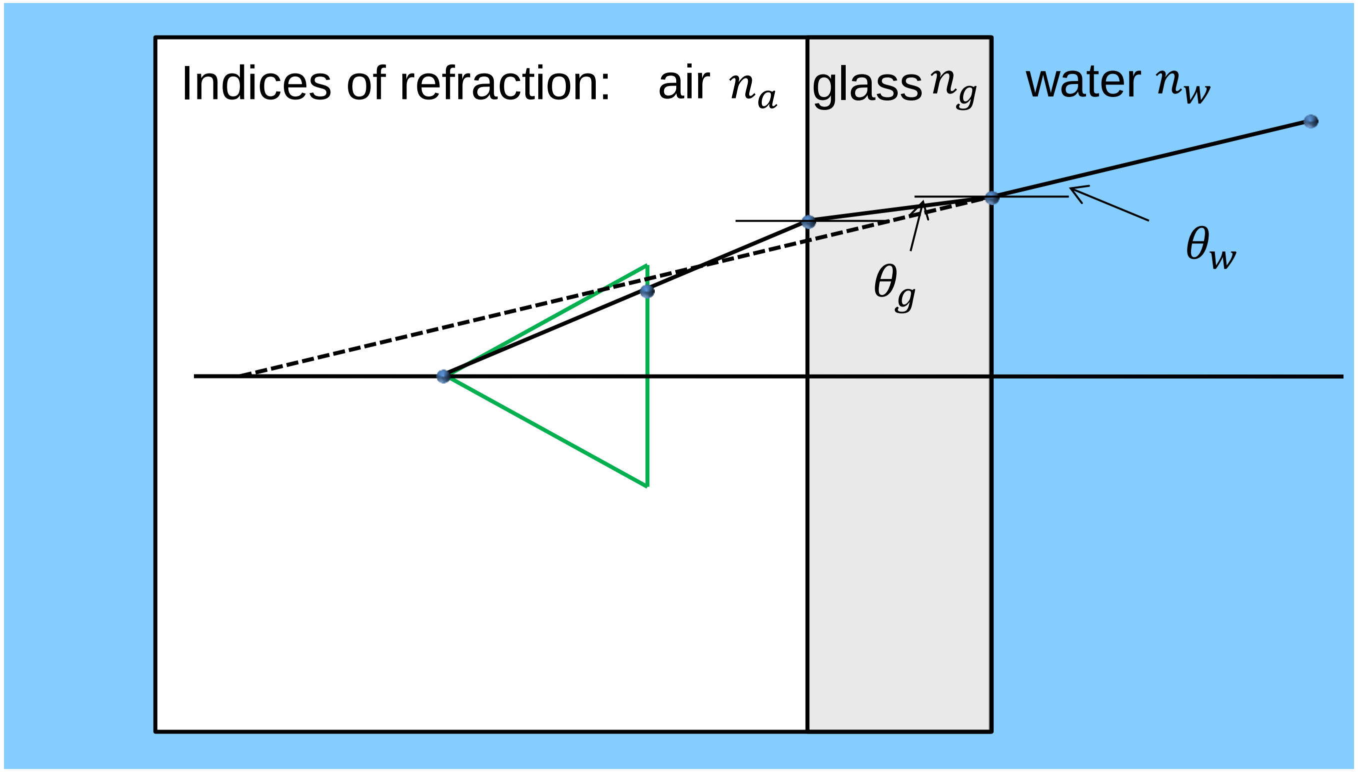


- different fields of view in air and water
- bent rays allow to “look around” objects to some extent
- changes in focus, especially for dome port cameras

Refraction Examples

Left: top view of turtle in fish tank. Right: refracted view through glass into water.

Refractive Camera Model (thick glass)



Snell's law:

$$\frac{\sin\theta_w}{\sin\theta_g} = \frac{n_g}{n_w}$$

Hecht 05

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Exemplary Indices of Refraction

Medium	Index of Refraction (n)
air (λ = 589 nm)	1.0003
pure **water** (λ = 700 nm, 30°C, depth ~ 0 m)	1.329
pure **water** (λ = 700 nm, 30°C, depth > 11 000 m)	1.343
sea **water** (λ = 700 nm, 30°C, depth ~ 0 m)	1.335
sea **water** (λ = 700 nm, 30°C, depth > 11 000m)	1.349
sea **water** (λ = 400 nm, 30°C, depth > 11 000m)	1.363
quartz **glass** (λ = 589 nm)	1.4584
acrylic **glass** (λ = 589 nm)	1.51

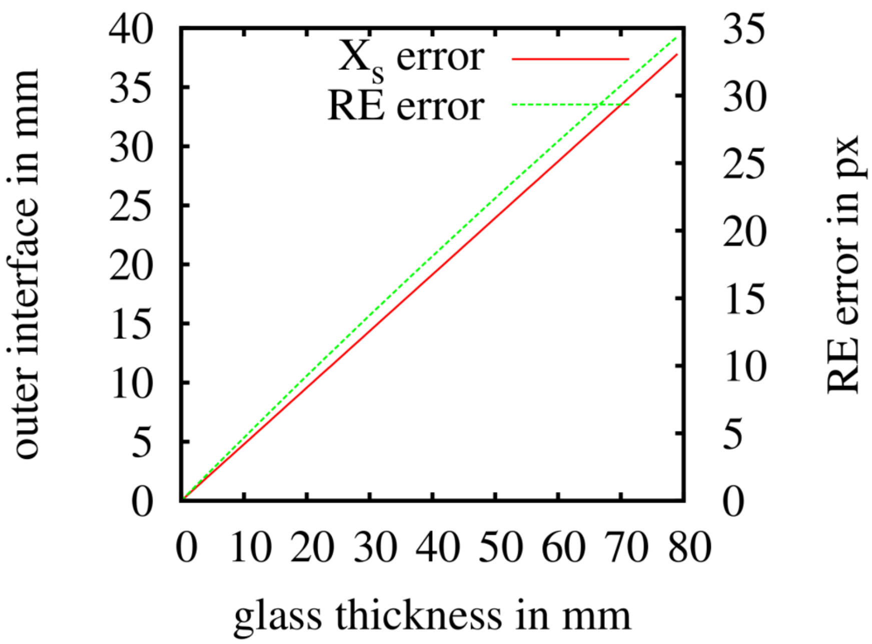
- index of refraction for water depends on water pressure, salinity, temperature and wavelength

Hecht 05, Mobley 1994

Ignoring Glass Thickness

camera:

- image size 1920x1080 px
- focal length 1080 px
- interface distance 10 mm
- interface normal $(0,0,1)^T$



- X_s error : point shift on outer interface after setting glass thickness to zero
- RE error: re-projection error after back-projecting to 3D point at 3000 mm distance and projecting with zero glass thickness

=> For pressure housings, glass must be considered !

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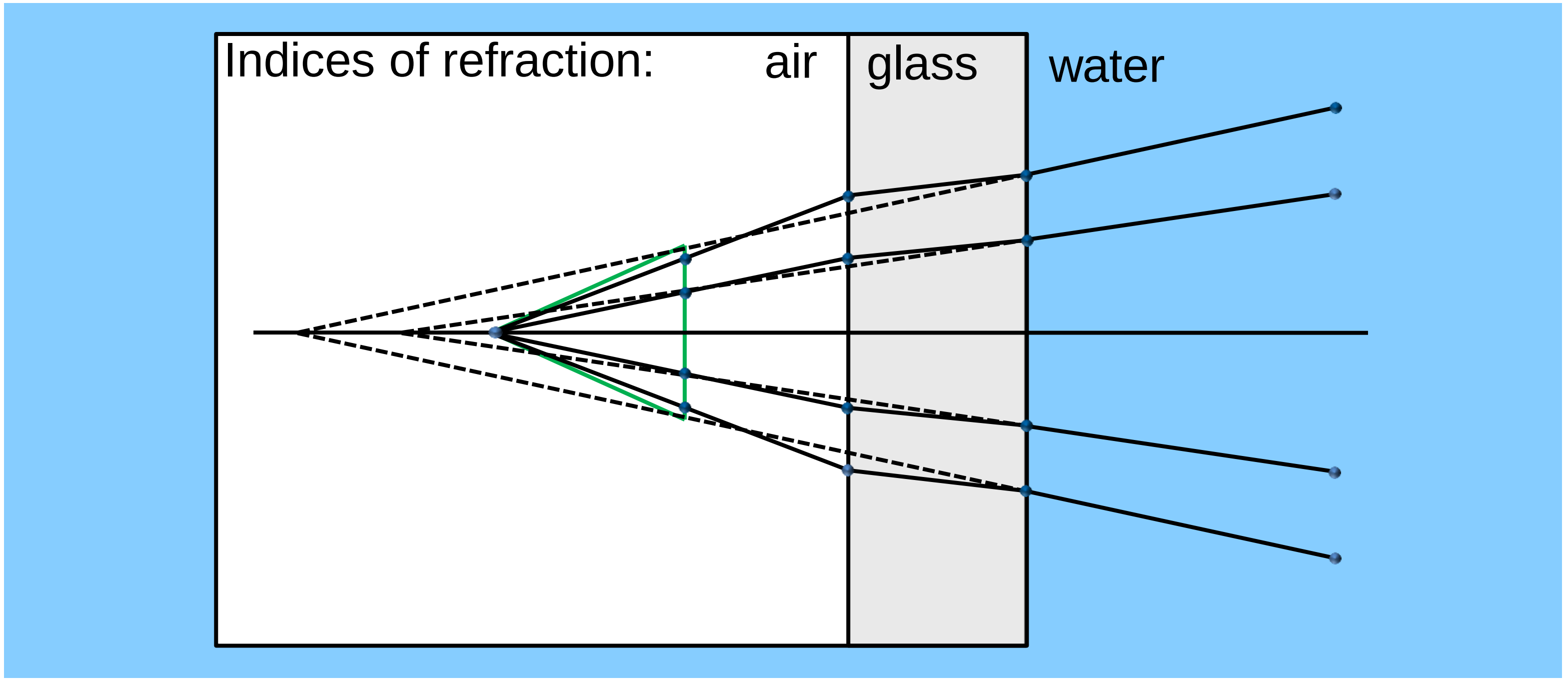
Refractive Camera Model



Main assumption of the pinhole camera model invalid!
Refraction needs to be modeled explicitly!

Treibitz and Schechner 08, Agrawal et al. 2012

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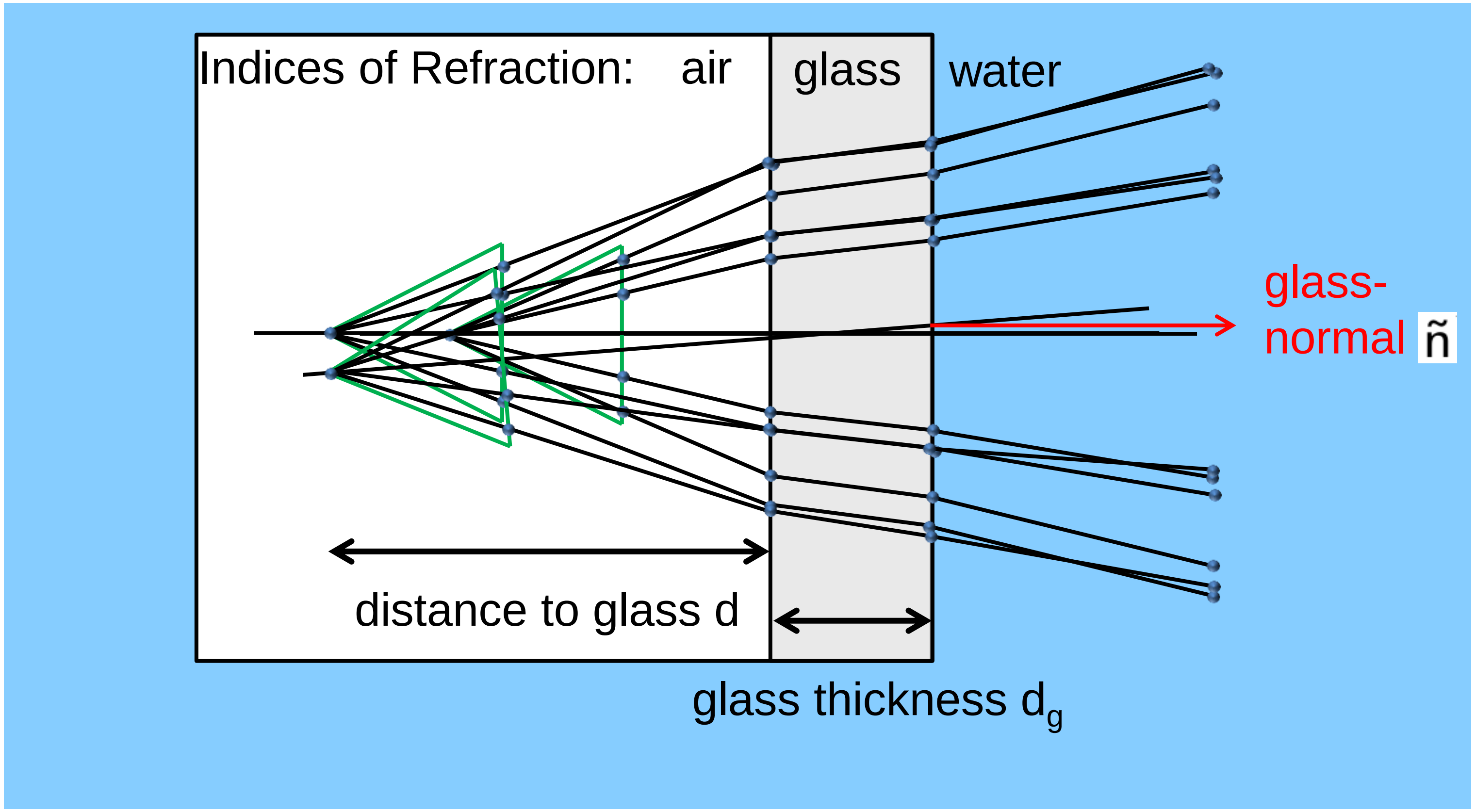
Parameterization of Refractive Camera



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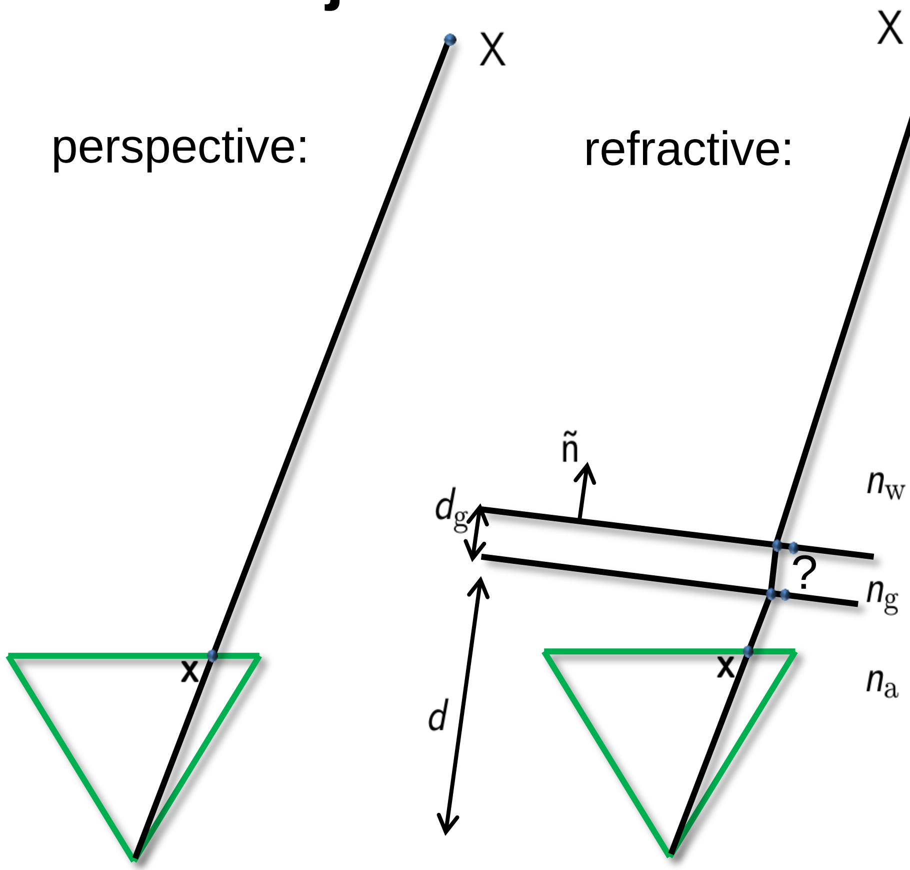
3D-2D Projection



3D-2D Projection:

- optimization (Kunz et al. 2008)
- determine roots of 12th degree polynomial (*Agrawal et al. 2012*)!
- glass thickness = 0, then the polynomial only has degree 4 (Glaeser/Schröcker 2001)

$x = P \cdot X$

$\mathscr{P}(x, X, \tilde{n}, d_g, d, n_a, n_g, n_w) = 0$

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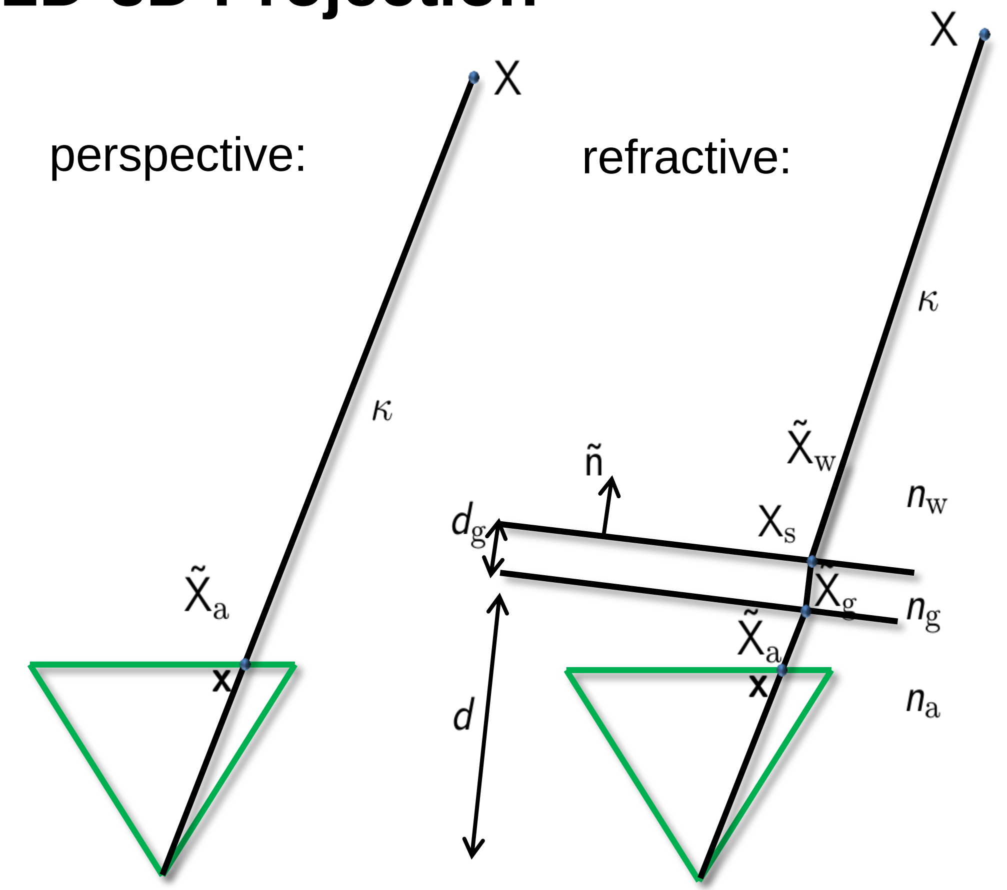
2D-3D Projection



$$\mathbf{x} = (x, y, 1)^T$$

$$(X_s, \tilde{X}_w) = \mathrm{Ray}(\mathbf{x}, d, \tilde{n})$$

$$X = X_s + \kappa \tilde{X}_w$$

ñ interface normal
d interface distance
d_g interface thickness

Glassner 1994

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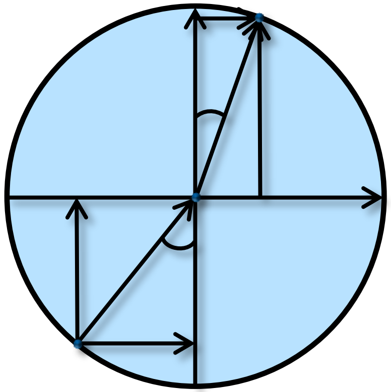
Ray Computation



Pixel $\mathbf{x} = (x, y, 1)^{\mathrm{T}}$

Ray in air: $\tilde{\mathrm{X}}_{\mathrm{a}} = \frac{\mathbf{K}^{-1}\mathbf{x}}{||\mathbf{K}^{-1}\mathbf{x}||_2}$

Ray in glass:

$$\mathrm{X}_{\mathrm{g}} = \underbrace{\frac{n_{\mathrm{a}}}{n_{\mathrm{g}}}}_{=:a} \tilde{\mathrm{X}}_{\mathrm{a}} + \underbrace{\left(-\frac{n_{\mathrm{a}}}{n_{\mathrm{g}}} \tilde{\mathrm{X}}_{\mathrm{a}}^{\mathrm{T}} \tilde{\mathrm{n}} + \sqrt{1 - \frac{n_{\mathrm{a}}}{n_{\mathrm{g}}} (1 - (\tilde{\mathrm{X}}_{\mathrm{a}}^{\mathrm{T}} \tilde{\mathrm{n}})^2)} \right)}_{=:b} \tilde{\mathrm{n}}$$

$$= a\tilde{\mathrm{X}}_{\mathrm{a}} + b\tilde{\mathrm{n}}$$

Normalized ray in glass: $\tilde{\mathrm{X}}_{\mathrm{g}} = \frac{\mathrm{X}_{\mathrm{g}}}{||\mathrm{X}_{\mathrm{g}}||_2}$

Starting point on outer glass plane: $\mathrm{X}_{\mathrm{s}} = \frac{d}{\tilde{\mathrm{X}}_{\mathrm{a}}^{\mathrm{T}} \tilde{\mathrm{n}}} \tilde{\mathrm{X}}_{\mathrm{a}} + \frac{d_{\mathrm{g}}}{\tilde{\mathrm{X}}_{\mathrm{g}}^{\mathrm{T}} \tilde{\mathrm{n}}} \tilde{\mathrm{X}}_{\mathrm{g}}$

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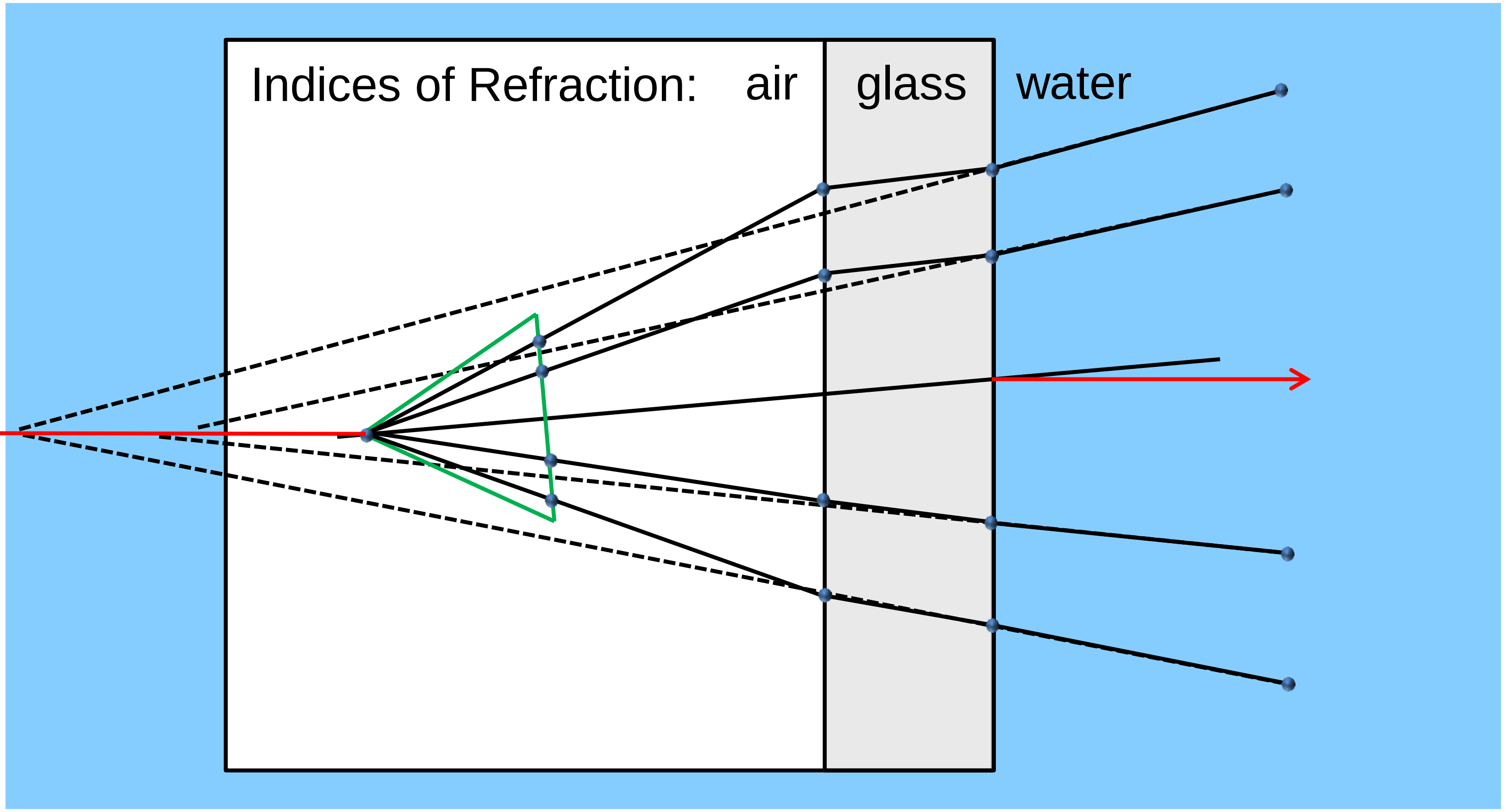
Axial Camera



All rays intersect common axis defined by interface normal and center of projection

Agrawal et al. 2012

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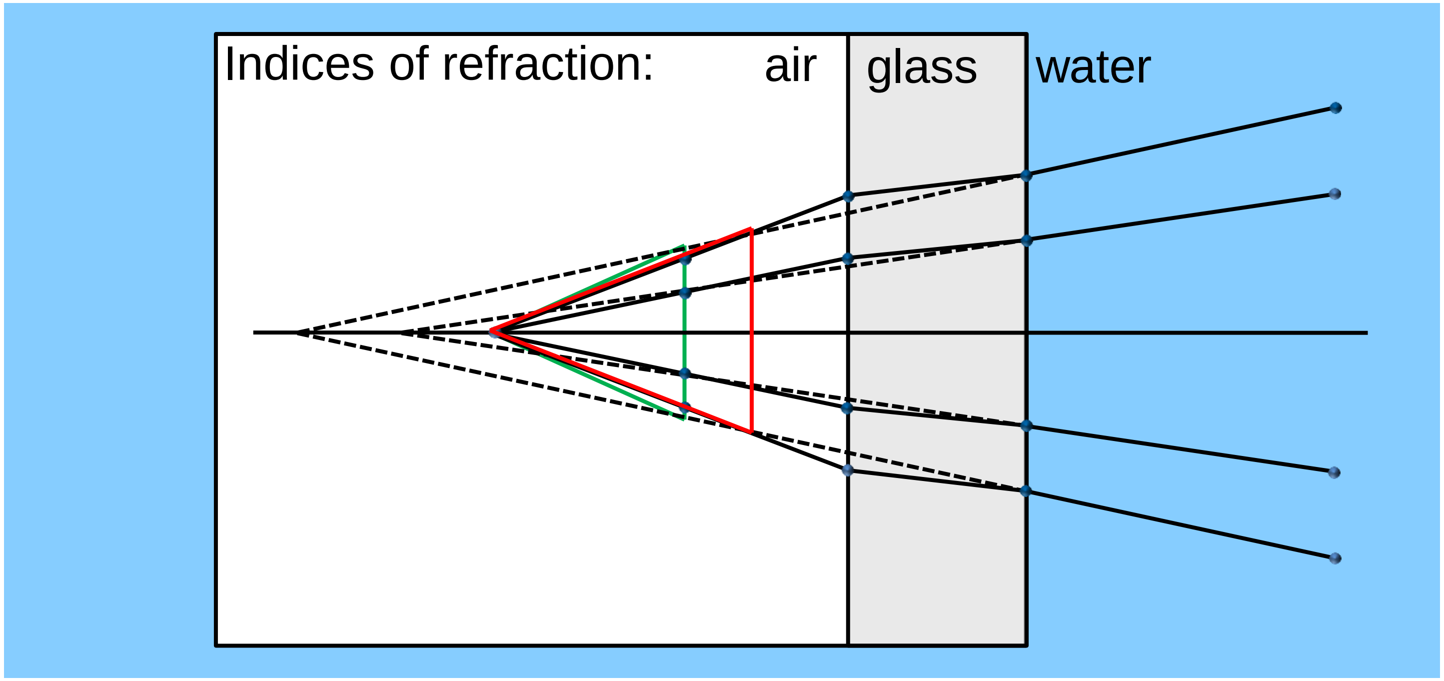
Approximating Refraction with Pinhole Camera Model



perspective approximation:
- focal length
- radial distortion
- principal point

Refraction can be approximated with the perspective camera model by allowing the intrinsics and extrinsics to absorb the effect to some extend

Lavest et al. 2000, Freyer and Fraser 1986, Sedlazeck and Koch 2012

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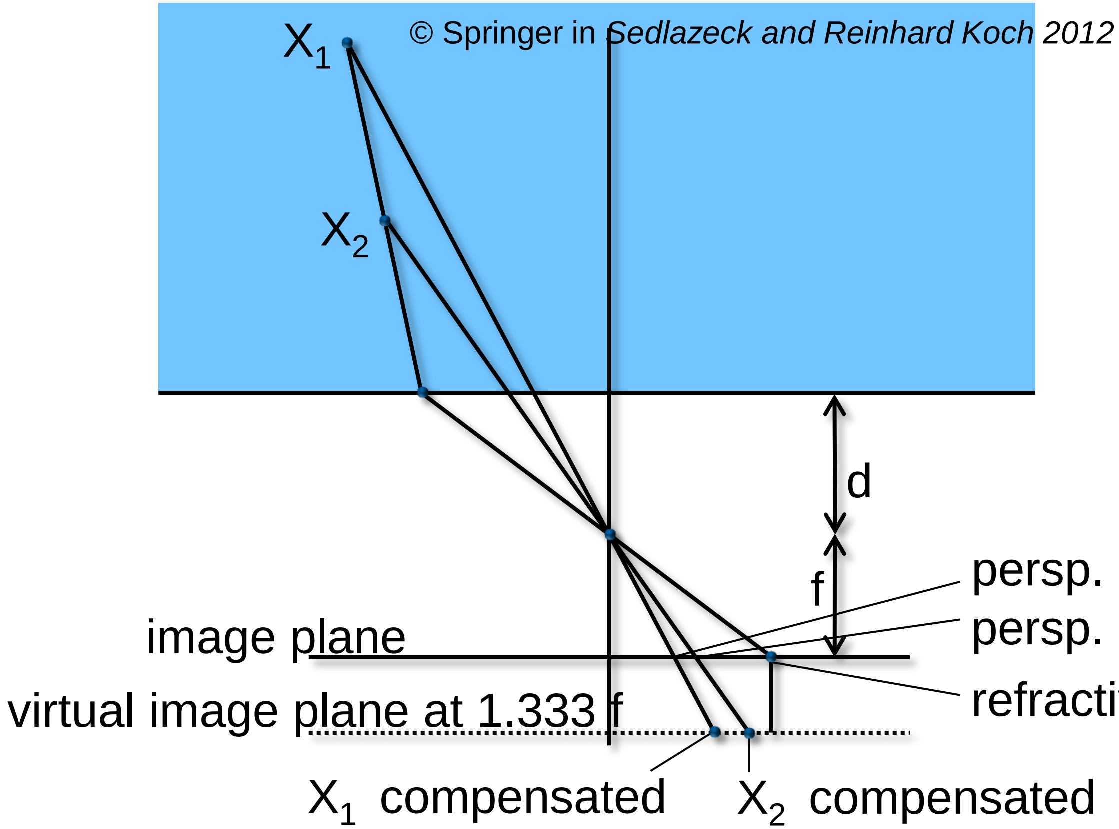
Distance Dependency of Error



Distance dependency of the error causes an error in the approximation because radial distortion does not depend on distance.

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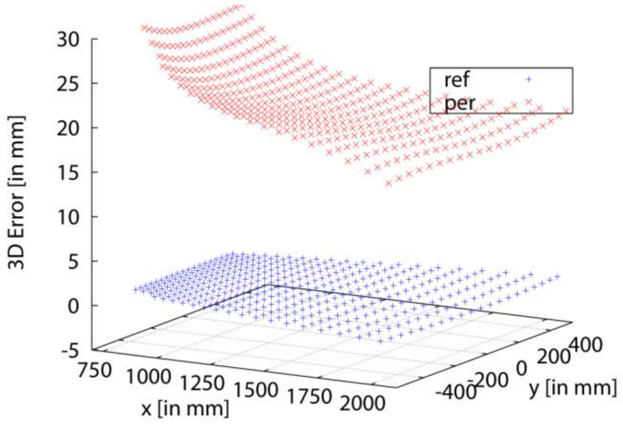
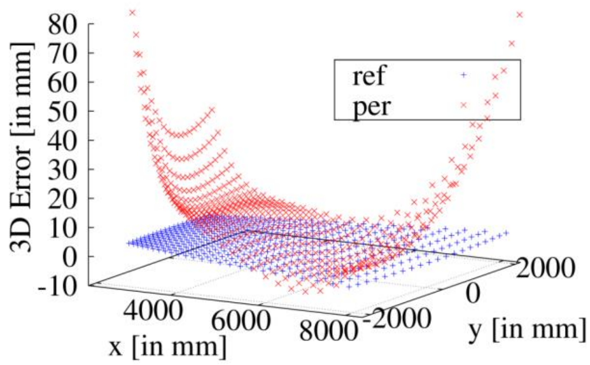
Perspective Projection Approximation Error



distance: 1000 mm



distance: 4000 mm

Plane Triangulation:
- refractive
- perspective

Triangulation errors for plane triangulation. In case of explicitly modeling refraction, the error is zero. The error of the perspective approximation depends on distance and pixel position.

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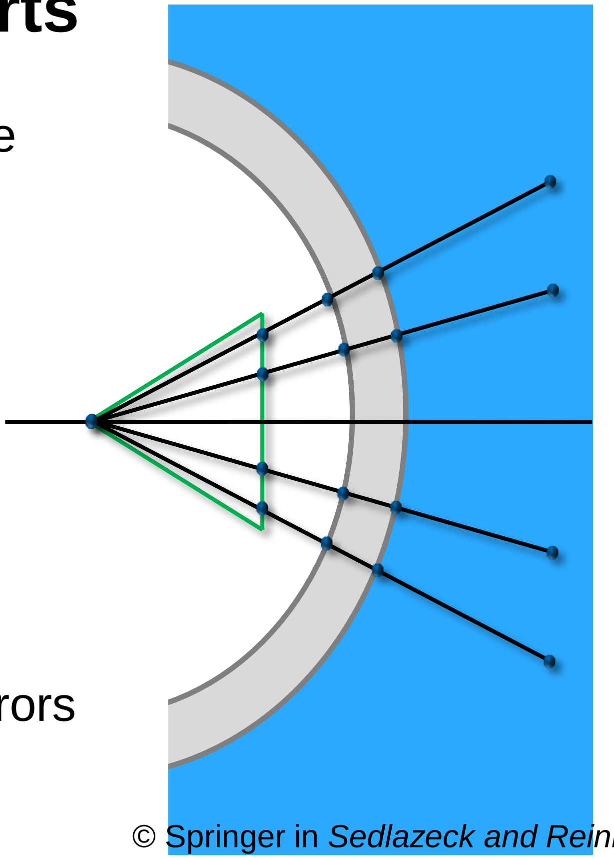
Interesting Alternative: Dome Ports

Spherical glass: single viewpoint in center of dome



© Springer in *Sedlazeck and Reinhard Koch 2012*

Challenges:
- More difficult to make, expensive
- nSVP and strong distortions for decentering errors
- error smaller compared to flat port
- dome acts as lens itself => focus issues

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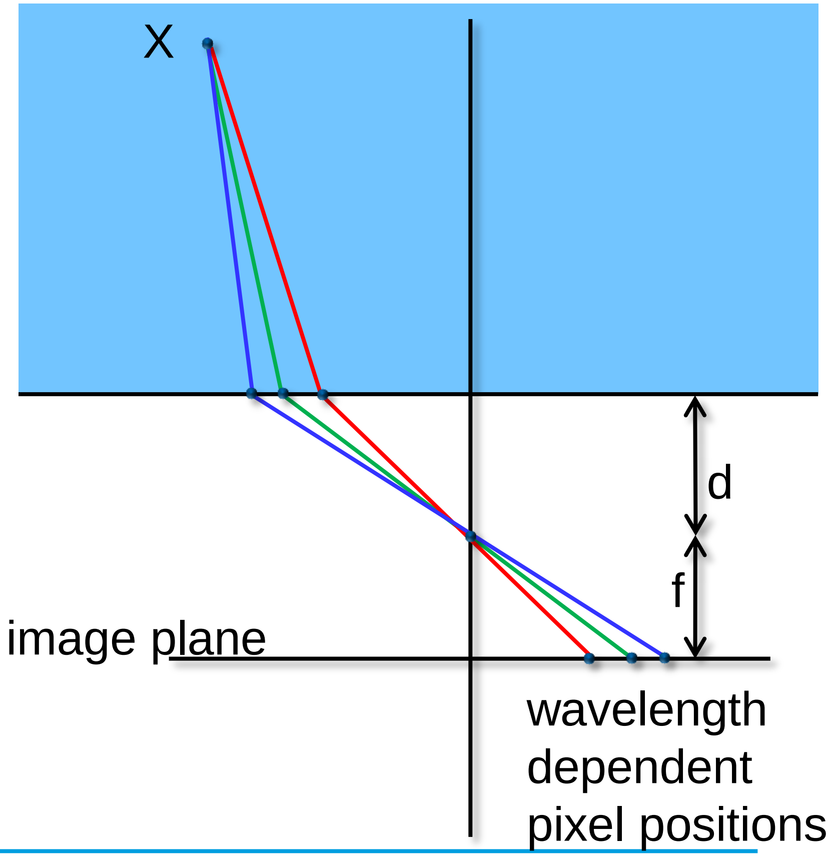
Wavelength Dependency

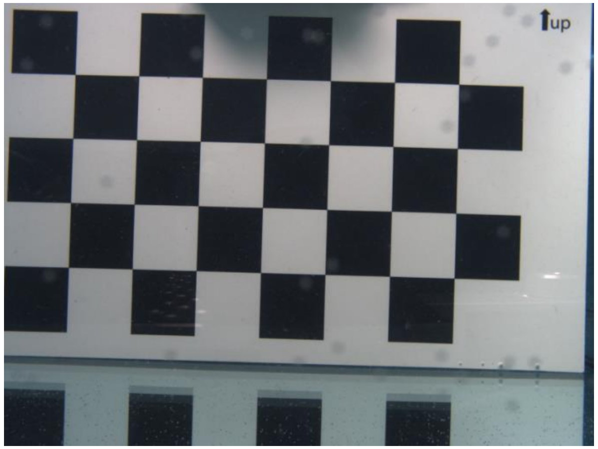
- indices of refraction are wavelength dependent -> different color channels
- can be observed in images
- *Yau et al.2013* use active calibration target with blue and red colors
- need to calibrate chromatic aberrations of lens
- helps to make calibration more robust

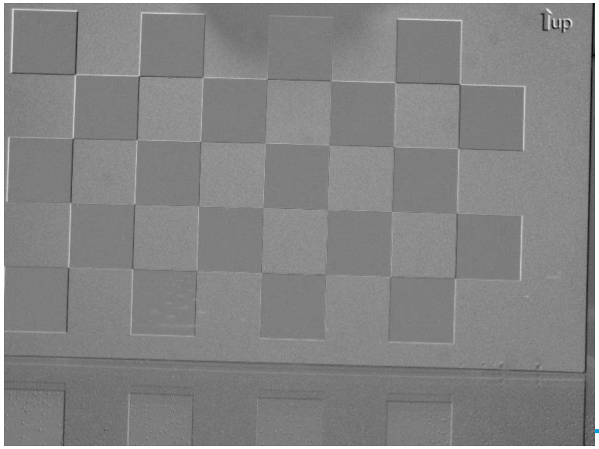
Wavelength	Index of refraction water
660 nm	1.33151
589 nm	1.33344
405 nm	1.34318

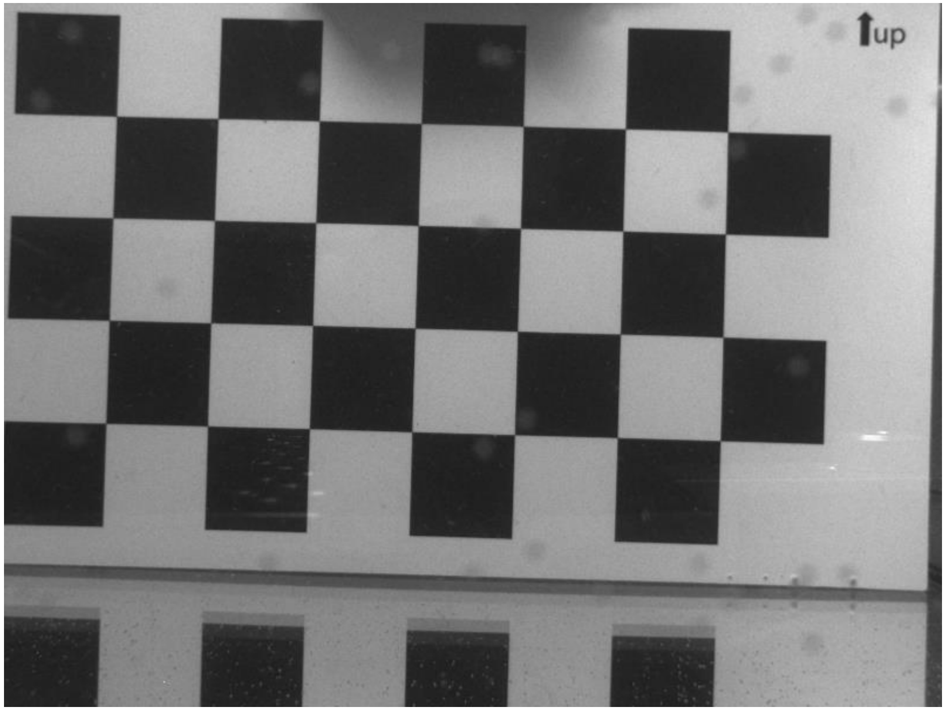


Yau et al. 2013

Wavelength Dependency







upper left: original image
lower left: difference image of red and blue channel
right: red and blue color channels

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Further Reading

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Wrap Up

- perspective camera model
- models imaging geometry with focal length, etc. and camera pose
- refractive camera for underwater case required
- refractive 3D-2D projection computationally expensive, but virtual camera error is efficient
- approximating refraction with perspective camera causes distance dependent, systematic modeling error