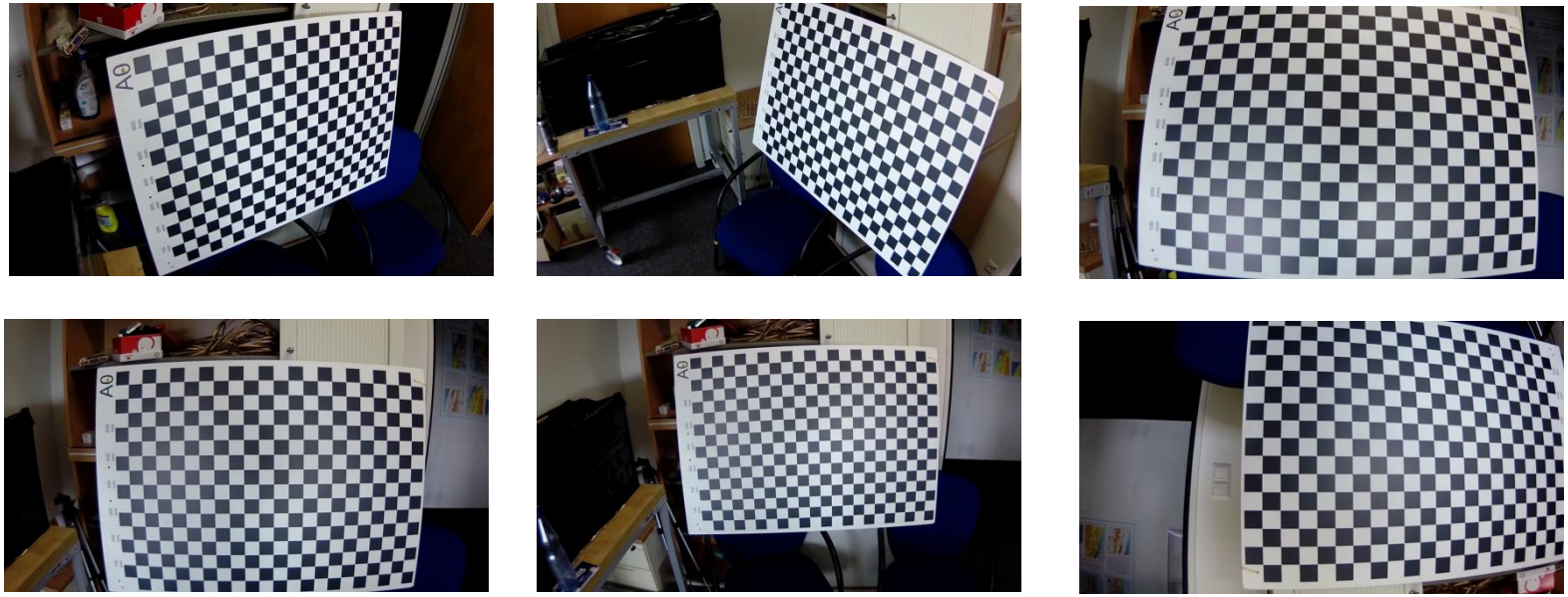
# Outline

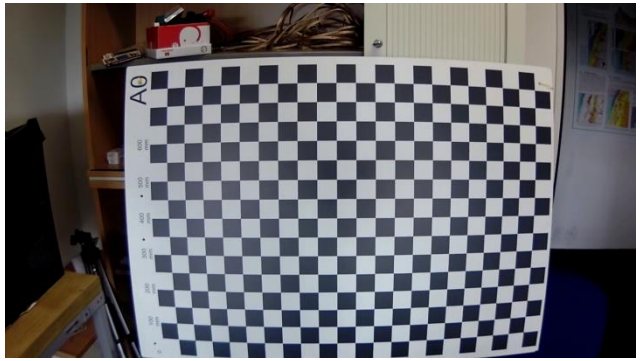
- Introduction
- Features and Feature Matching
- Geometry of Image Formation
- **Calibration**
- Structure from Motion
- Dense Stereo
- Conclusion

## Perspective Camera Calibration



- capture checkerboard images from different points of view
- detect corners in images (2D-3D correspondences)
- utilize set of known, planar 3D points for calibration

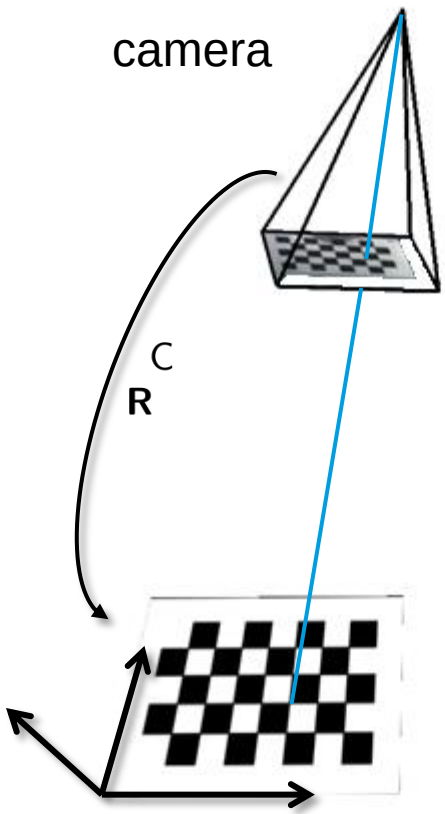
## Perspective Camera Calibration




- determine intrinsics (shared by all images):
  - focal length
  - principal point
  - aspect ratio
  - distortion parameters

- determine extrinsics (for each image):
  - rotation
  - translation

## Perspective Camera Calibration




Initialization:

- $\mathbf{x} = \mathbf{P}\mathbf{X}$

- Direct Linear Transform (DLT) of projection equation

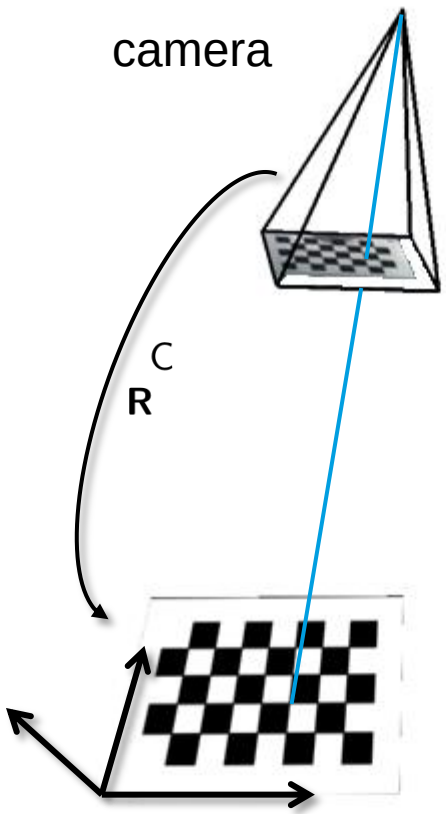
$$[\mathbf{x}]_\times \mathbf{P}\mathbf{X} = 0$$

- set of 2D-3D correspondences yields linear system of equations
- solve with SVD for entries in P

- decompose $\mathbf{P} = \left(\underbrace{\mathbf{K}\mathbf{R}^{\mathrm{T}}} \,\middle|\, -\mathbf{K}\mathbf{R}^{\mathrm{T}}\mathrm{C}\right)$ with RQ decomposition

*Heikkila and Silven 1997*

## Perspective Camera Calibration




Optimization:

- non-linear projection equation with distortion:

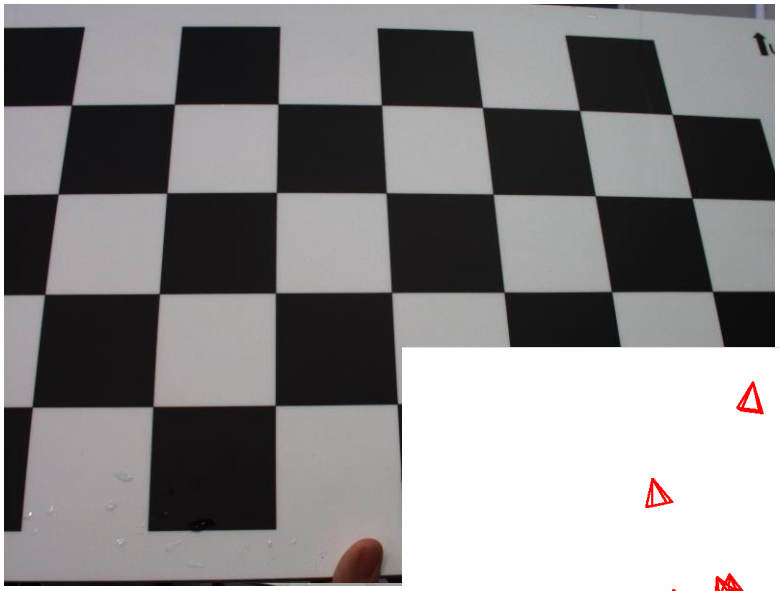
$$\mathrm{argmin}_{\forall \mathbf{R}_i, \mathbf{C}_i, i \in \{1,\ldots,N\}, f, c_\mathrm{x}, c_\mathrm{y}, k_1, k_2} \sum_{i=1}^{N} \sum_{j=1}^{M} \| \mathrm{proj}(\mathbf{X}_j, f, c_\mathrm{x}, c_\mathrm{y}, k_1, k_2, \mathbf{R}_i, \mathrm{C}_i) - \mathbf{x}_{ij} \|_2^2$$

- minimize using Levenberg Marquardt algorithm

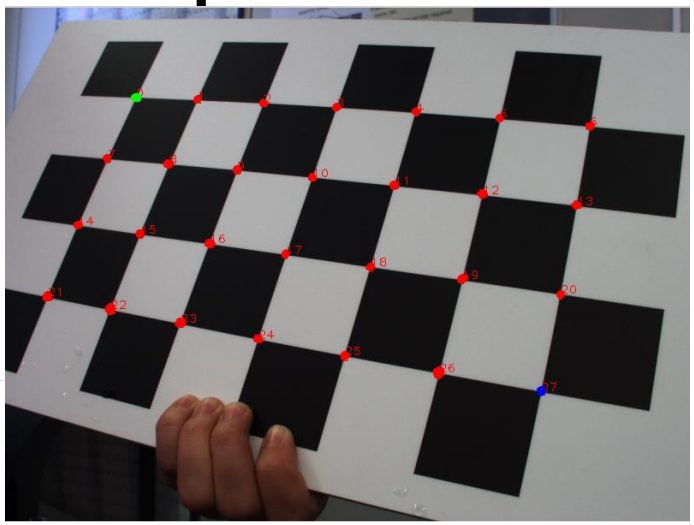
*Heikkila and Silven 1997*

GEOMAR

## Perspective Camera Calibration - Example



Corner Detection



Initialization, Optimization

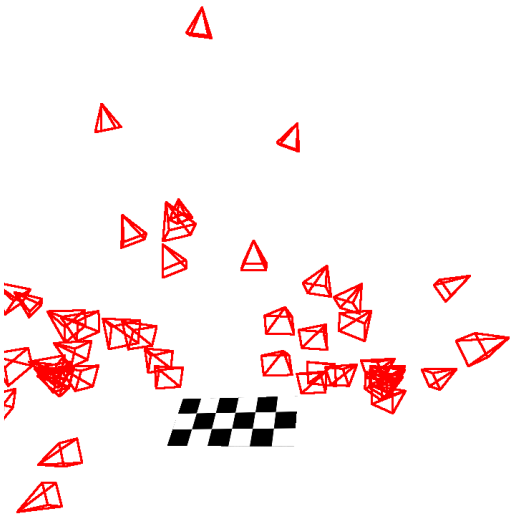


image size: 800x600 px
focal length: 911 px
principal point: (402,316)
rad dist: -0.2393, 0.1820
error init: 0.47
error opt: 0.37





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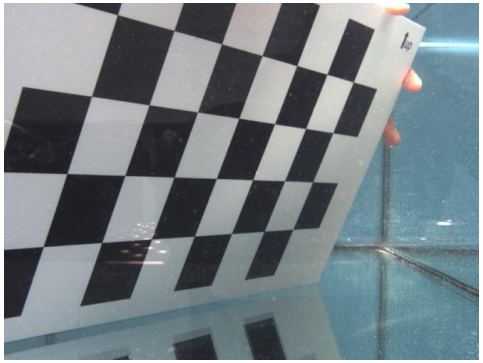
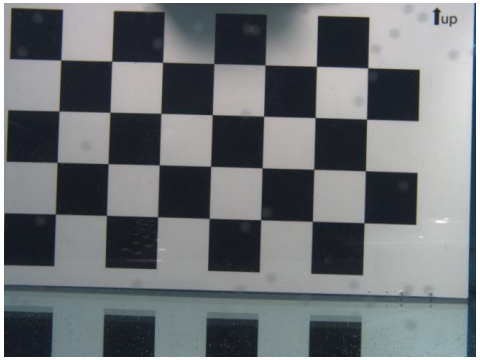
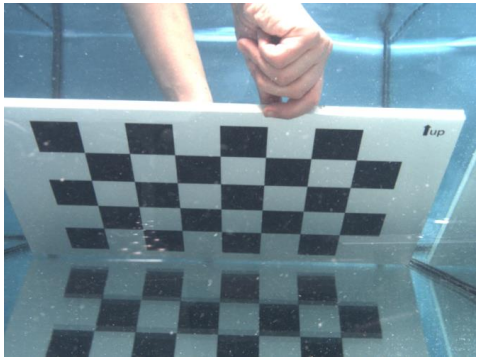
# Calibration of Refractive Camera Model

- intrinsic parameters: *Schiller et al. ISPRS 2008*
- input images from different view points
- Initialization: method by *Agrawal et al., CVPR 2012*
- Analysis-by-Synthesis method for optimization by *Jordt-Sedlazeck et al., ECCV 2012*







## Initialization

Given
- flat port underwater cameras are axial cameras
- assume camera‘s intrinsics are known, i.e. focal length, principal point and distortion parameters
- use 1 checkerboard image (with j*k 2D-3D correspondences from checkerboard detector)

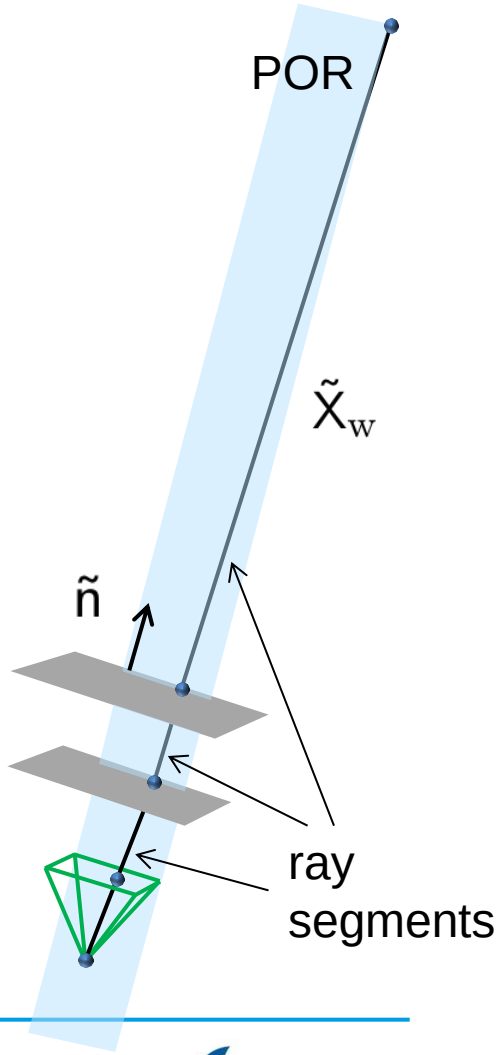
Estimate:
- **interface distance and tilt**, unknown camera pose

Geometric Constraints:
- “Plane of Refraction” (uses parallelism of interfaces)
- “Flat Refractive Constraint” (path consistency)

*Agrawal et al. 2012*

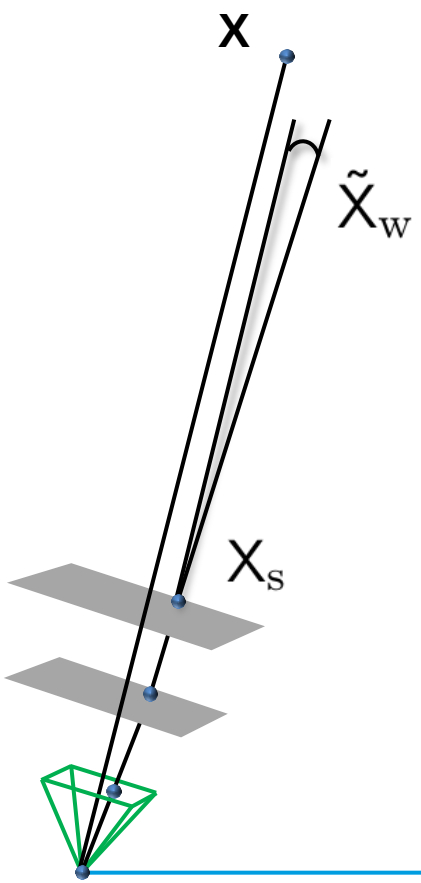
## Plane of Refraction (POR) Constraint

- all ray segments and the normal $\tilde{n}$ lie in common plane (second part of Snell's law) Plane of Refraction
- POR: $(\tilde{n} \times \tilde{X}_a)$
- 3D point transformed into camera coordinate system must lie in this plane as well: $(\mathbf{R}^T X - \mathbf{R}^T C)^T (\tilde{n} \times \tilde{X}_a) = 0$
- stacking equations derived from POR to linear system of equations and solve
- yields normal, camera rotation, and camera translation perpendicular to axis




*Agrawal et al. 2012*

## Flat Refractive Constraint (FRC)




- FRC is then used to determine layer thickness, i.e. interface distance and glass thickness
- result is used in non-linear optimization

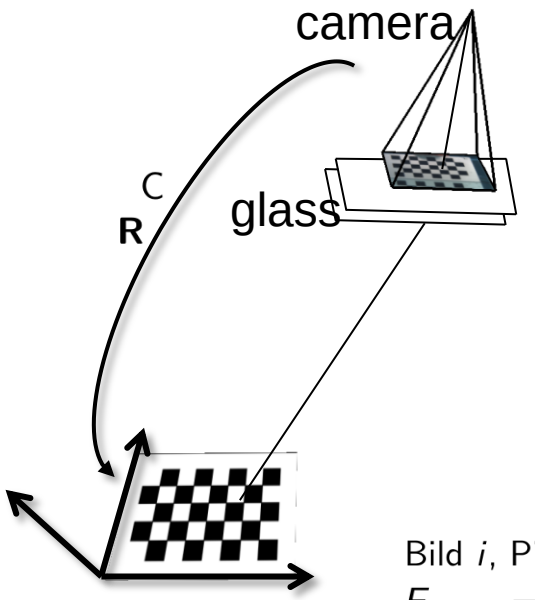
$$\mathbf{R}^T X - \mathbf{R}^T C$$

*Agrawal et al. 2012*

## Analysis-by-Synthesis Optimization

- results from Agrawal‘s calibration routine yield interface distance for each image
- further optimization enforces shared glass port parameterization for all images
- Analysis-by-Synthesis optimization is independent of errors in corner detection

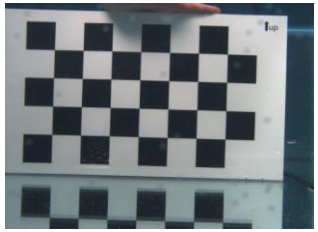
# Analysis-by-Synthesis Calibration




$$(\mathsf{X}_{\mathrm{s}}^{\mathrm{cc}}, \tilde{\mathsf{X}}_{\mathrm{w}}^{\mathrm{cc}}) = \mathrm{Ray}(\mathbf{x}, d, \tilde{\mathsf{n}})$$

$$(\mathsf{X}_{\mathrm{s}}^{\mathrm{wc}}, \tilde{\mathsf{X}}_{\mathrm{w}}^{\mathrm{wc}}) = (\mathbf{R}\mathsf{X}_{\mathrm{s}}^{\mathrm{cc}} + \mathsf{C}, \mathbf{R}\tilde{\mathsf{X}}_{\mathrm{w}}^{\mathrm{cc}})$$

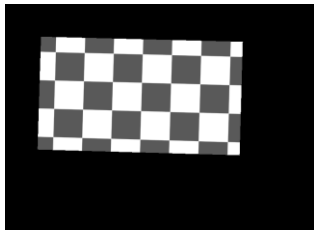
$$\mathsf{X} = \mathsf{X}_{\mathrm{s}}^{\mathrm{wc}} + \kappa \tilde{\mathsf{X}}_{\mathrm{w}}^{\mathrm{wc}}$$



input image

pixel color on checkerboard

$$I_{\mathrm{ren}}(\mathsf{x})$$

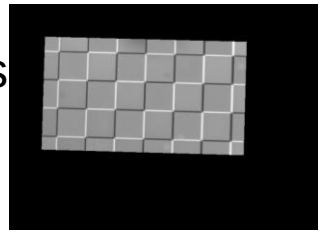


synthetic image

error across **all** pixels and images

Bild $i$, Pixel $k$

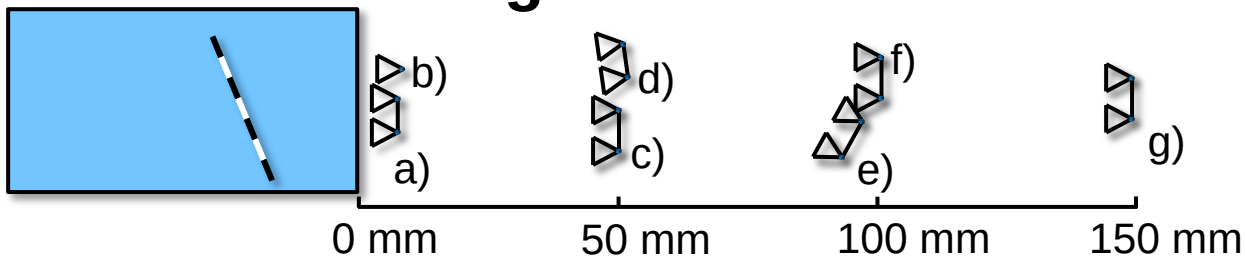
$$E_{\mathrm{AbyS}} = \sum_{i<N} \sum_{k<K} (I_{\mathrm{ren}}(\mathsf{x}_{ik}) - I(\mathsf{x}_{ik}))^2$$



difference image (error)

non-linear optimization with CMA-ES

# Calibration Results on Real Images




| | | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|---|
| monocular: left camera | $d_1$ in mm | 7.88 | 10.60 | 51.95 | 50.53 | 86.25 | 95.54 | 149.97 |
| | $\theta_1$ in ° | 0.34 | 0.25 | 0.29 | 8.06 | 29.29 | 0.12 | 0.12 |
| monocular: right camera | $d_2$ in mm | 8.07 | | 44.47 | 47.30 | 89.67 | 99.60 | 150.0 |
| | $\theta_2$ in ° | 0.29 | | 0.23 | 7.97 | 29.16 | 0.29 | 0.39 |
| rig: left camera | $d_1$ in mm | 19.38 | | 48.07 | 45.30 | 79.96 | 100.39 | 147.13 |
| | $\theta_1$ in ° | 0.27 | | 0.11 | 7.99 | 28.40 | 0.22 | 0.05 |
| rig: right camera | $d_2$ in mm | 0.72 | | 58.82 | 28.34 | 102.81 | 113.16 | 160.47 |
| | $\theta_2$ in ° | 0.28 | | 0.42 | 7.79 | 27.79 | 0.89 | 0.09 |

## References

*Z. Zhang. Flexible Camera Calibration by Viewing a Plane from Unknown Orientations. ICCV99*

*J. Heikkilä and O. Silven. A Four-step Camera Calibration Procedure with Implicit Image Correction. CVPR97*

*I. Schiller, C. Beder, and R. Koch. Calibration of a PMD camera using a planar calibration object together with a multi-camera setup. In The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, volume Vol. XXXVII. Part B3a, pages 297–302, Beijing,China, 2008. XXI. ISPRS Congress.*

*A. Agrawal, S. Ramalingam, Y. Taguchi, and V. Chari. A theory of multi-layer flat refractive geometry. In CVPR, 2012.*

## References

*A. Jordt-Sedlazeck and R. Koch. Refractive calibration of underwater cameras. In Computer Vision - ECCV 2012, volume 7576 of Lecture Notes in Computer Science, pages 846–859. Springer Berlin Heidelberg, 2012.*

*N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. Evolutionary Computation, 9(2):159–195, 2001.*

## Further Reading

*R. Li, H. Li,W. Zou, R. G. Smith, and T. A. Curran. Quantitative photogrammetric analysis of digital underwater video imagery. Oceanic Engineering, IEEE Journal of, 22(2):364 –375, 4 1997.*

## Software

*Bouguet's Matlab Toolbox:*
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#ref

Opencv software: opencv.org

Kiel University (MIP):
http://www.mip.informatik.uni-kiel.de/tiki-index.php?page=Calibration

## Wrap Up

- perspective calibration based on checkerboard images yields:
  - intrinsics (focal length, principal point, radial distortion)
- extension on underwater images for refractive camera model:
  - housing parameters (interface distance, interface tilt)
- analysis-by-synthesis approach is independent of errors in corner detection