Outline

- Introduction
- Features and Feature Matching
- Geometry of Image Formation
- Calibration
- **Structure from Motion**
- Dense Stereo
- Conclusion

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Underwater 3D Reconstruction

Perspective	Refractive
approximate refraction with pinhole model	no systematic modeling error
established methods for pose estimation	require special methods for pose estimation
established methods for bundle adjustment (fast)	minimizing the reprojection error is infeasible, require virtual camera error and non-standard BA
established methods for dense depth estimation and 3D model computation	no established method can be directly applied to 3D model computation

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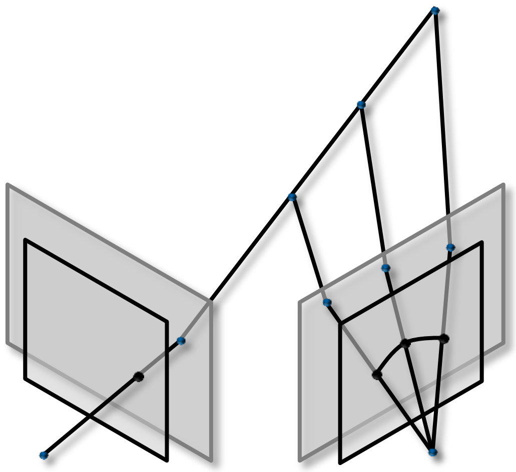
Refractive Relative Pose Estimation

- perspective relative pose estimation **not applicable**
- epipolar lines become curves under refraction

- linear method with Generalized Epipolar Constraint (GEC)
- iterative method



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Generalized Epipolar Constraint (GEC)

Pless 2003 developed generalized epipolar constraint (GEC) for nSVP cameras:

Plücker lines

$$\mathsf{M} = \tilde{\mathsf{X}}_{\mathrm{w}} \times \mathsf{X}_{\mathrm{s}} \qquad \mathsf{L} = (\tilde{\mathsf{X}}_{\mathrm{w}}, \mathsf{M})$$

$$\mathsf{M}' = \tilde{\mathsf{X}}'_{\mathrm{w}} \times \mathsf{X}'_{\mathrm{s}} \qquad \mathsf{L}' = (\mathbf{R}\tilde{\mathsf{X}}'_{\mathrm{w}}, \mathbf{R}\mathsf{M}' - [\mathsf{C}]_{\times} \mathbf{R}\tilde{\mathsf{X}}'_{\mathrm{w}})$$

Plücker line intersection iff:

$$0 = \tilde{\mathsf{X}}_{\mathrm{w}}^{\mathrm{T}} (\mathbf{R}\mathsf{M}' - [\mathsf{C}]_{\times} \mathbf{R}\tilde{\mathsf{X}}'_{\mathrm{w}}) + \mathsf{M}^{\mathrm{T}} (\mathbf{R}\tilde{\mathsf{X}}'_{\mathrm{w}})$$

$$= \begin{pmatrix} \tilde{\mathsf{X}}_{\mathrm{w}} \\ \mathsf{M} \end{pmatrix}^{\mathrm{T}} \underbrace{\begin{pmatrix} -[\mathsf{C}]_{\times} \mathbf{R} & \mathbf{R} \\ \mathbf{R} & \mathbf{0}_{3\times 3} \end{pmatrix}}_{\mathbf{E}_{\mathrm{GEC}}} \begin{pmatrix} \tilde{\mathsf{X}}'_{\mathrm{w}} \\ \mathsf{M}' \end{pmatrix}$$

Generalized Epipolar Constraint (GEC)

Pless 2003

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Refractive Relative Pose from GEC

- 17 GEC equations can be stacked to linear system of equations *(Pless 2003)* with 18 entries for **E** and **R:**

$$0 = \begin{pmatrix} \tilde{X}_{\mathrm{w}} \\ M \end{pmatrix}^{\mathrm{T}} \underbrace{\begin{pmatrix} -[C]_{\times}\mathbf{R} & \mathbf{R} \\ \mathbf{R} & \mathbf{0}_{3\times 3} \end{pmatrix}}_{\mathbf{E}_{\mathrm{GEC}}} \begin{pmatrix} \tilde{X}'_{\mathrm{w}} \\ M' \end{pmatrix}$$

- *Li et al. 2008:* no unique solution in case of axial cameras
- problem: R is already contained in E
- solution: solve for **E** in:

$$(\mathbf{A}_{\mathrm{R}}\mathbf{A}_{\mathrm{R}}^{+} - \mathbf{I})\mathbf{A}_{\mathrm{E}} \begin{pmatrix} e_{11} \\ \dots \\ e_{33} \end{pmatrix} = 0$$

Pless 2003, Li et al. 2008

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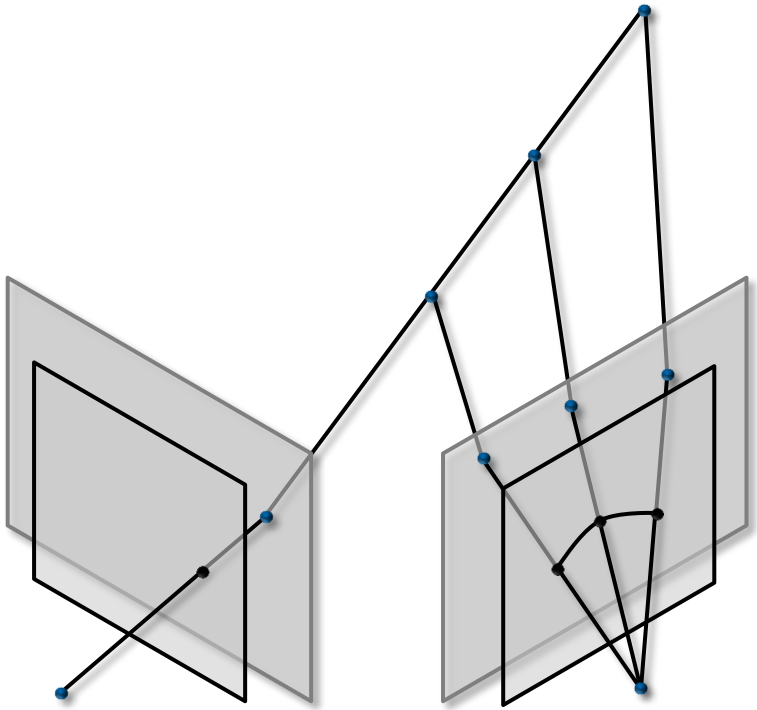
Iterative Refractive Relative Pose Estimation

- new method based on 3 geometric constraints:
  1. **Triangulation constraint**
  2. **FRC constraint** *(Agrawal 2012)*
  3. **POR constraint** *(Agrawal 2012)*



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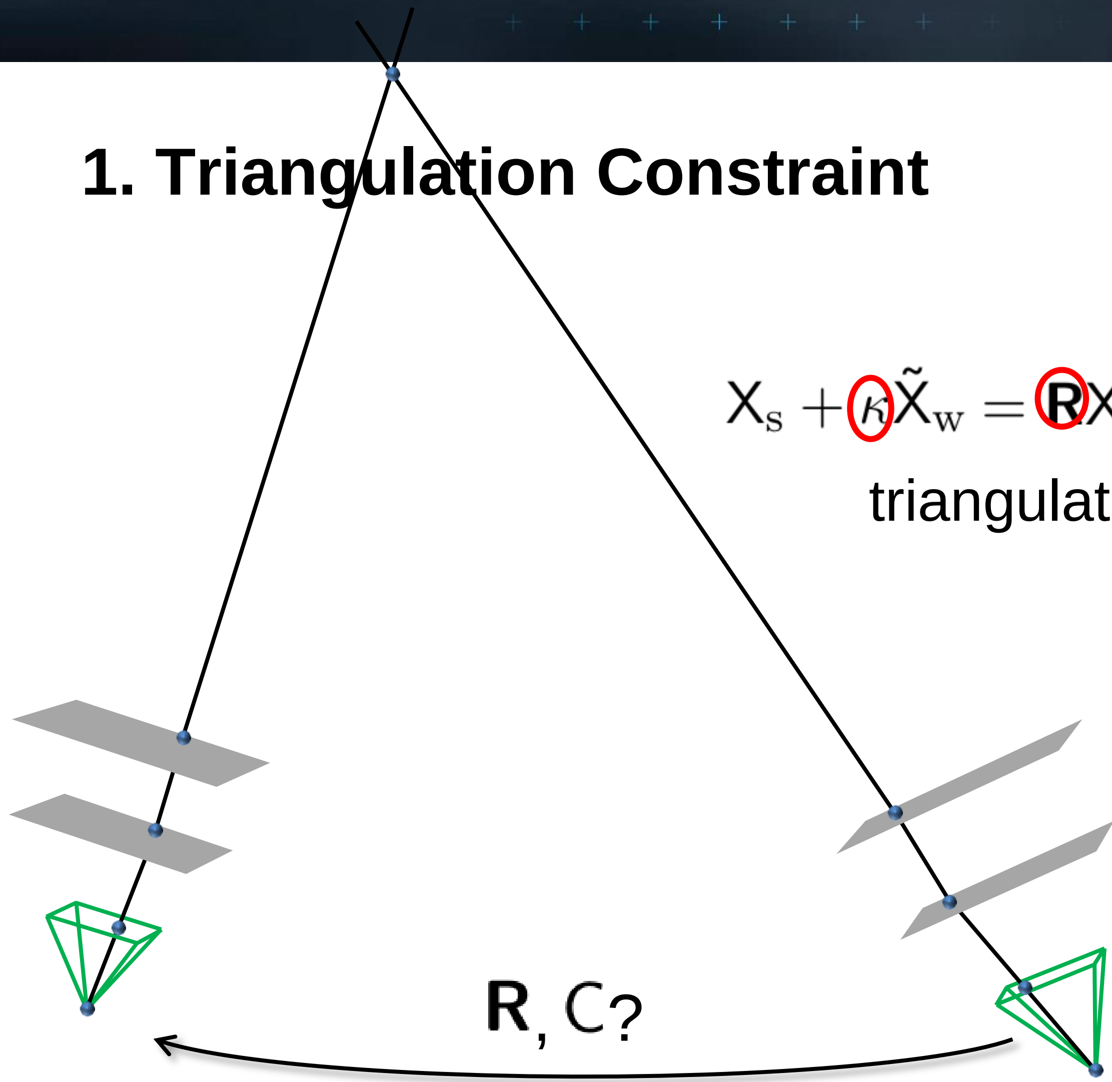
1. Triangulation Constraint



$$X_s + \kappa \tilde{X}_w = \mathbf{R} X'_s + C + \kappa' \mathbf{R} \tilde{X}_w, \quad \kappa, \kappa' \in \mathbb{R}$$

triangulation constraint

○ non-linear in the unknowns!

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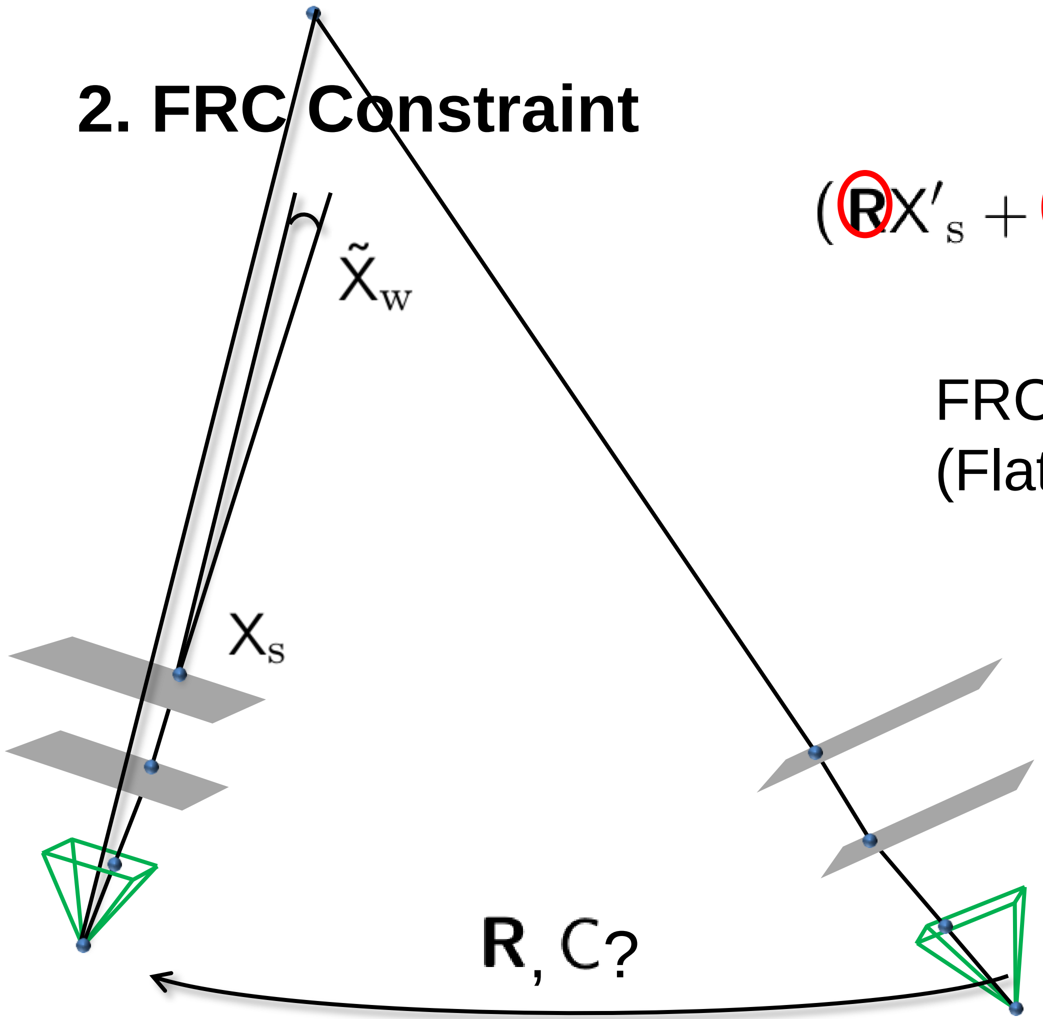
2. FRC Constraint



$$(RX'_s + C + \kappa' R\tilde{X}'_w - X_s) \times \tilde{X}_w = 0$$

FRC Constraint
(Flat Refractive Constraint)

O non-linear in the un-kowns!

FRC: Agrawal 2012

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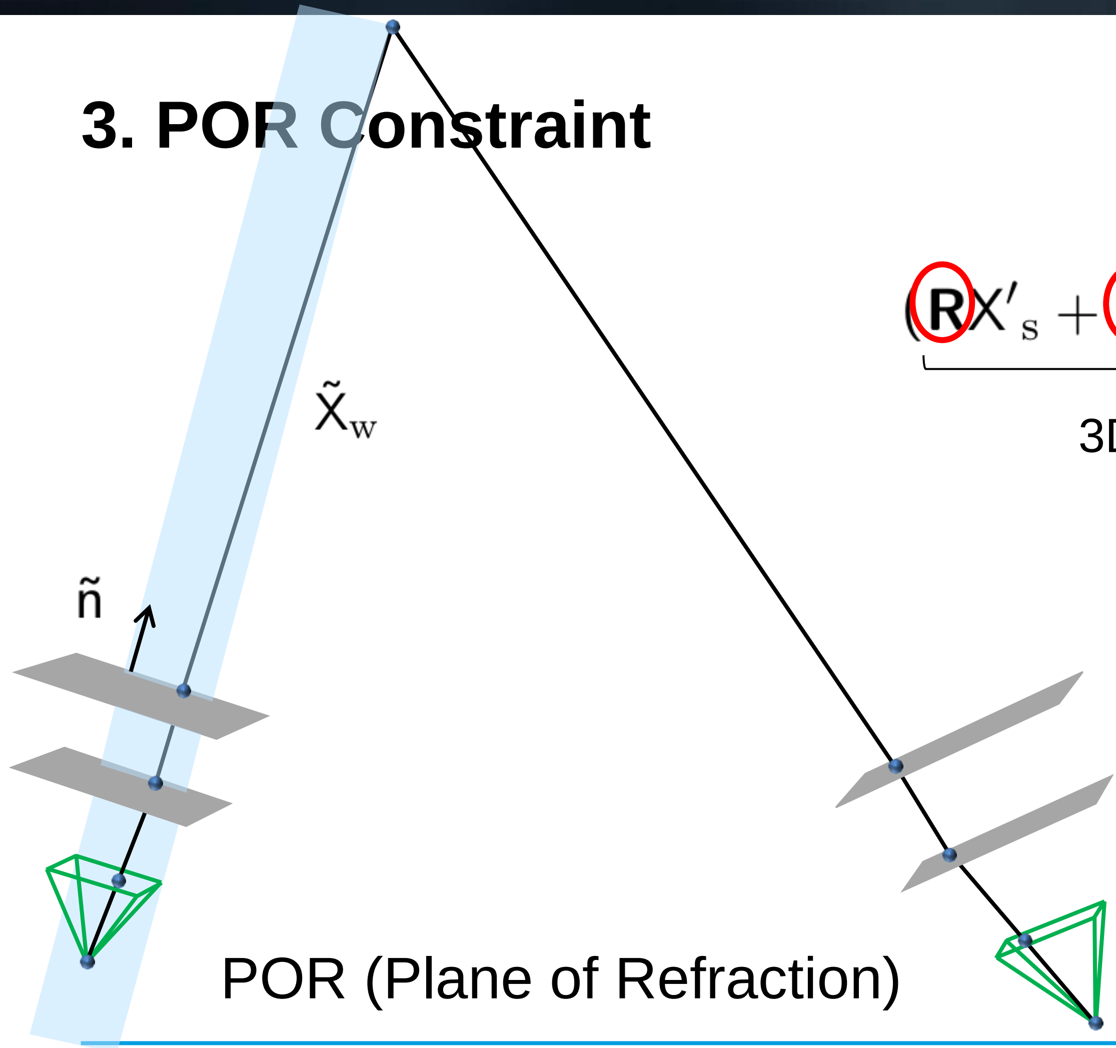
3. POR Constraint

$$\underbrace{(\mathbf{R}X'_{s} + C + \kappa'\mathbf{R}\tilde{X}'_{w})}_{\text{3D point}}{}^{T}\underbrace{(\tilde{n} \times \tilde{X}_{w})}_{\text{POR}} = 0$$



O non-linear in the un-kowns!

POR: Agrawal 2012

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Relative Poses Estimation – Iterative Method

$\kappa^0 = 3000, \kappa'^0 = 3000 \; \forall k \in \{1, ..., K\}$

$$\begin{bmatrix} \text{Triang. Constraint}(\kappa^\nu, \kappa'^\nu)_1 \\ \text{FRC-Constraint}(\kappa^\nu, \kappa'^\nu)_1 \\ \vdots \\ \text{Triang. Constraint}(\kappa^\nu, \kappa'^\nu)_K \\ \text{FRC-Constraint}(\kappa^\nu, \kappa'^\nu)_K \end{bmatrix} \begin{bmatrix} r_{11} \\ r_{12} \\ \vdots \\ r_{33} \\ C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_{2K} \end{bmatrix}$$

$\mathbf{R}^{\nu+1}, \mathrm{C}^{\nu+1}$

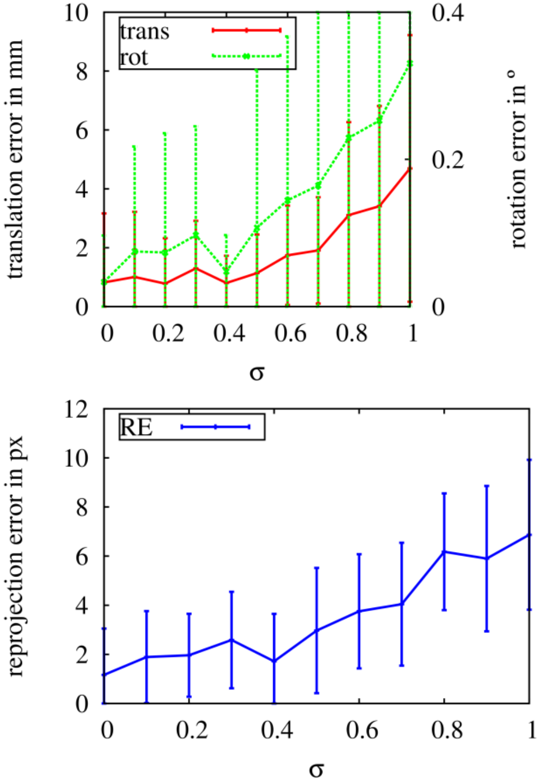
$$\begin{matrix} \text{POR-Constraint}(\mathbf{R}^{\nu+1}, \mathrm{C}^{\nu+1})_1 \\ \vdots \\ \text{POR-Constraint}(\mathbf{R}^{\nu+1}, \mathrm{C}^{\nu+1})_K \end{matrix}$$

$\kappa_k^{\nu+1}, \kappa'^{\nu+1}_k \; \forall k \in \{1, ..., K\}$

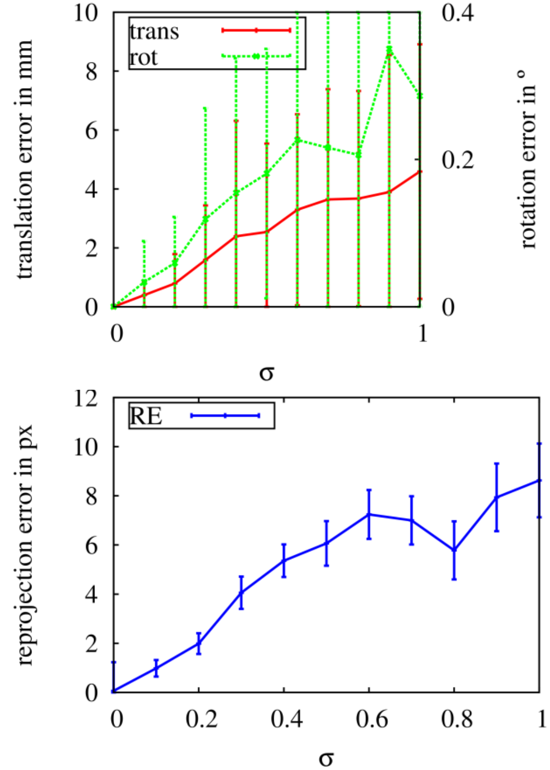
Jordt-Sedlazeck 2013

Relative Poses Estimation - Results

- non-linear optimization with virtual camera error RANSAC Algorithm for outliers in correspondences
- scene size: ca. 3000 mm
- error comparable to perspective results on perspective data



iterative method



linear method (GEC)

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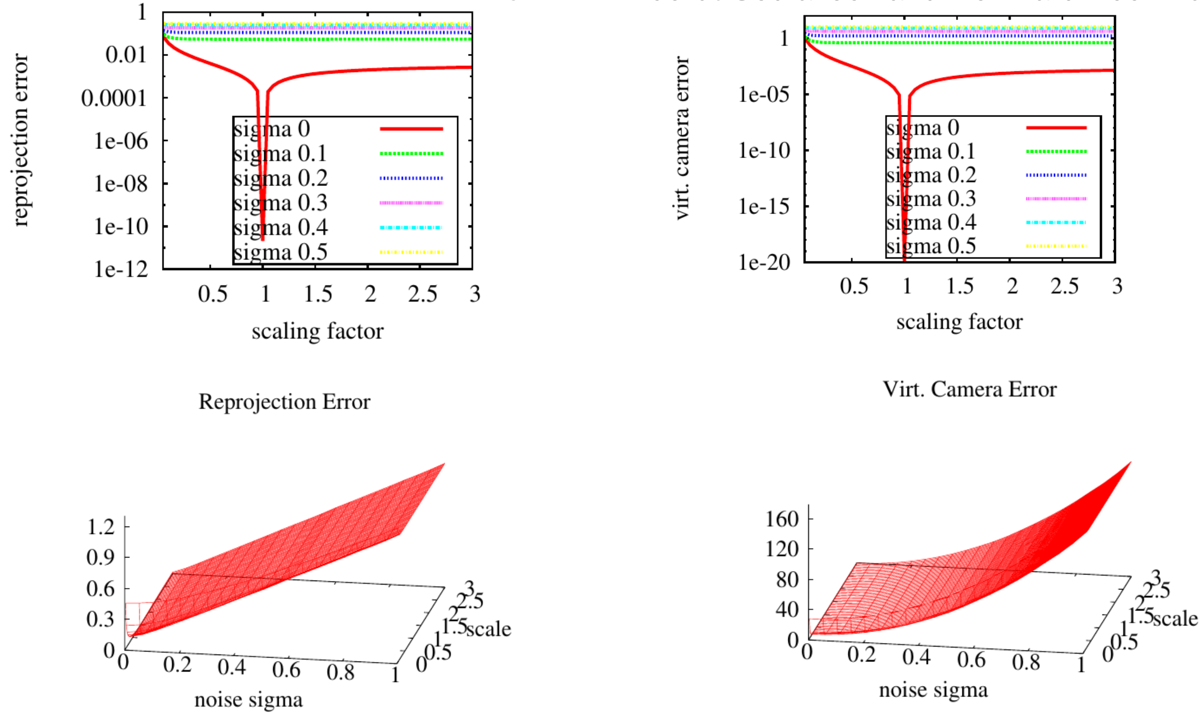
Metric Scaling

Note that refractive relative pose estimation is based on 3D rays and their starting points, those are metric!

- scene scale can be retrieved theoretically
- only works for zero noise on correspondences
- plots show two-view reprojection error and virtual camera error depending on baseline scale



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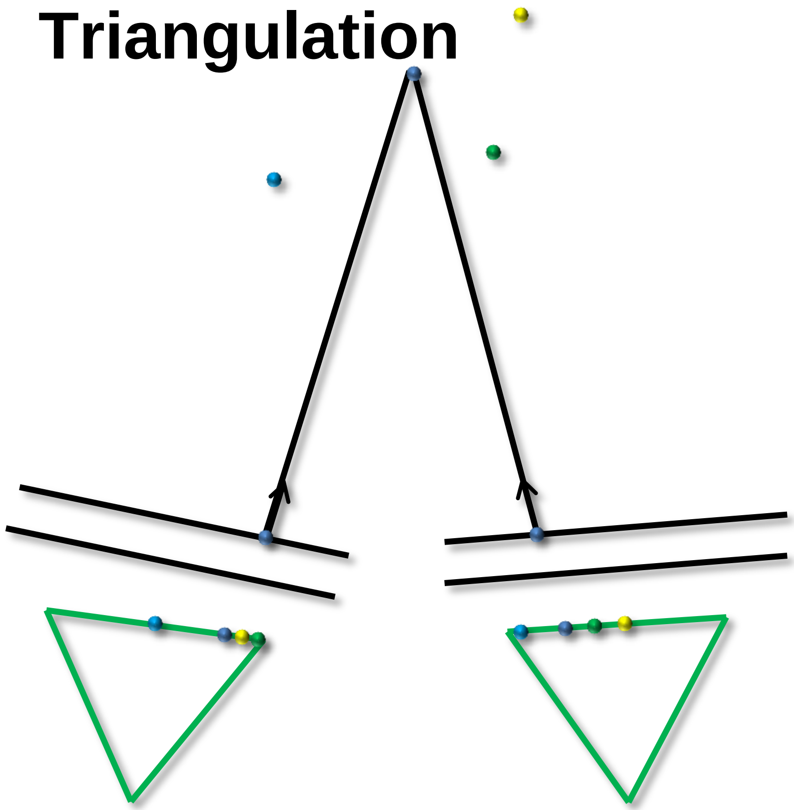
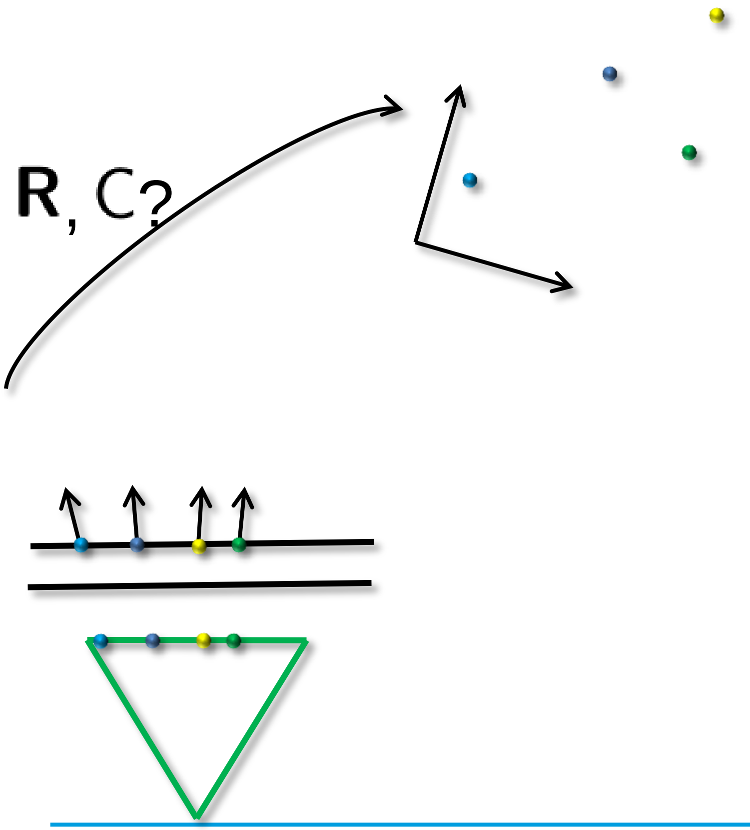
Triangulation



Result:
- second camera pose relative to first
- 3D points for all 2D-2D correspondences

Hartley and Sturm: Triangulation 1997

Refractive Absolute Pose Estimation



$$X_k = \mathbf{R} X_{s_k} + C + \kappa_k \mathbf{R} \tilde{X}_{w_k}$$

○ non-linear in the un-knowns!

similar, iterative method as in case of relative pose estimation

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Bundle Adjustment

	Perspective	Refractive
error function	reprojection error	??
constraint type	f(p)=l (explicit)	??
optimization scheme	Gauss Markov	??
derivatives	analytic	??

- ➢ reprojection error in refractive case infeasible

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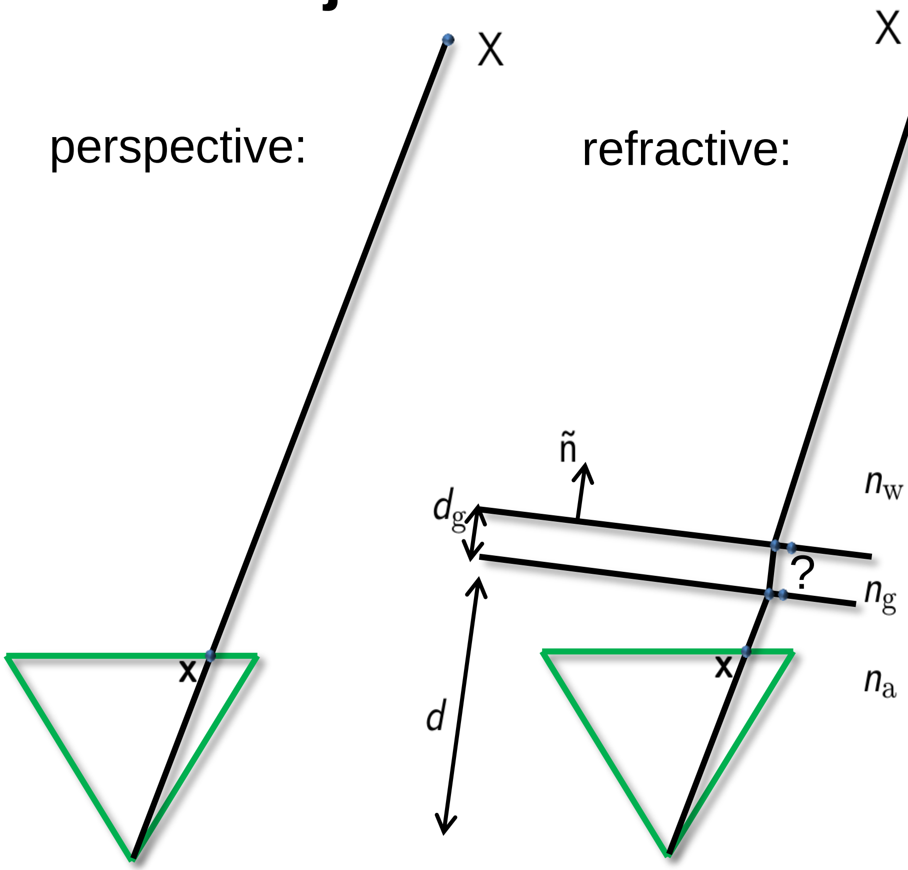
3D-2D Projection



3D-2D Projection:

- optimization (Kunz et al. 2008)
- determine roots of 12th degree polynomial (*Agrawal et al. 2012*)!
- glass thickness = 0, then the polynomial only has degree 4 (Glaeser/Schröcker 2001)

$x = P \cdot X$ (*f(p)=l*) $\mathcal{P}(\tilde{x}, X, n, d_g, d, n_a, n_g, n_w) = 0$ (*g(l,p)=0*)

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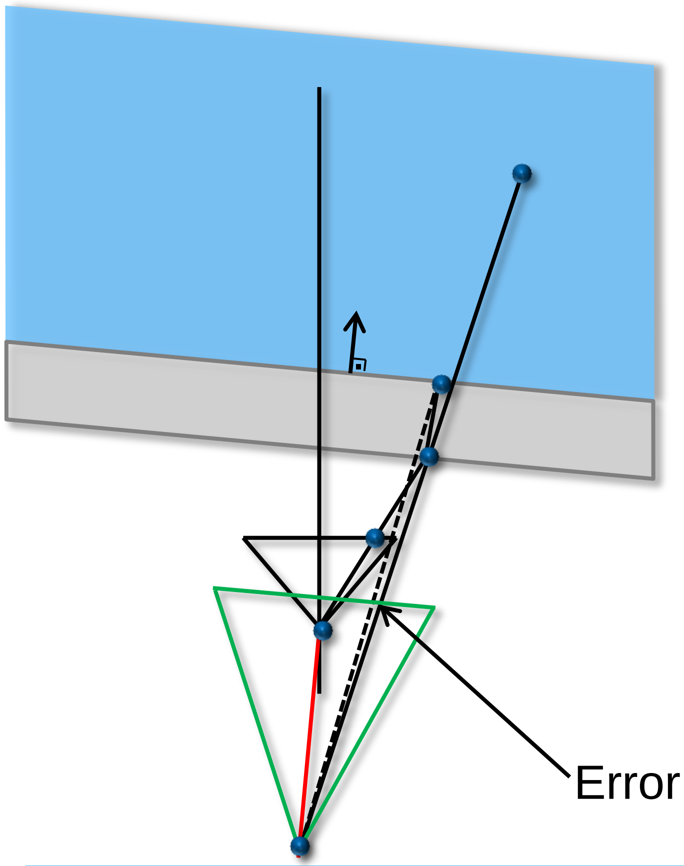
Virtual Camera Error and non-linear Optimization



Non-Linear Error Function

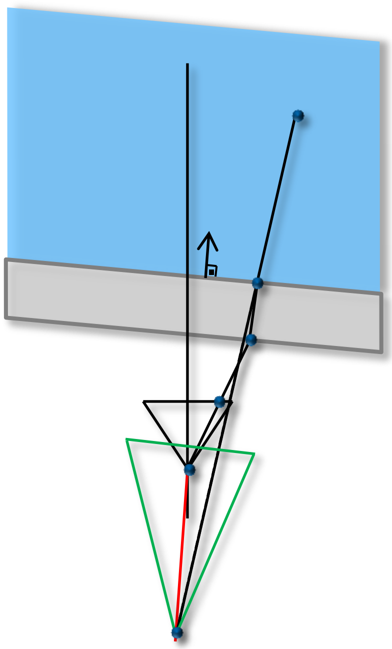
- Definition of a virtual camera for each 2D pixel allows perspective projection of 3D points into virtual camera
- **More efficient than** 3D-2D projection
- **Analytic derivatives** can be computed

Jordt-Sedlazeck und Koch 2013

Bundle Adjustment

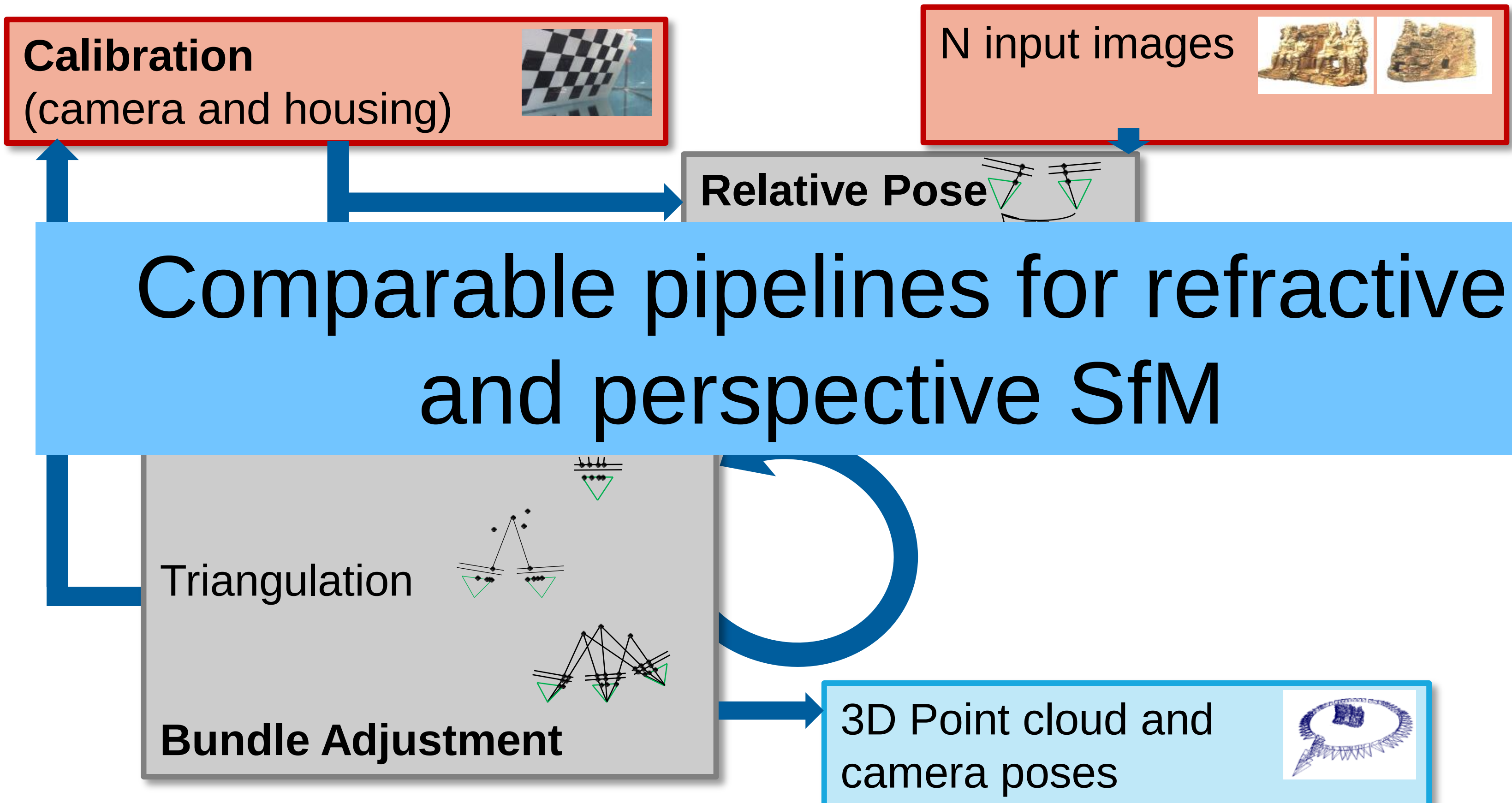
	Perspective	Refractive
error function	reprojection error	virtual camera error
constraint type	f(p)=l (explicit)	g(p,l)=0 (**implicit!**)
optimization scheme	Gauss Markov	Gauss Helmert
derivatives	analytic	analytic

- reprojection error in refractive case infeasible
- run-time reduction of factor 1000 compared to using reprojection error



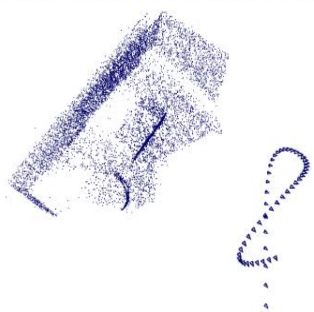
virtual camera error

Structure from Motion - Overview

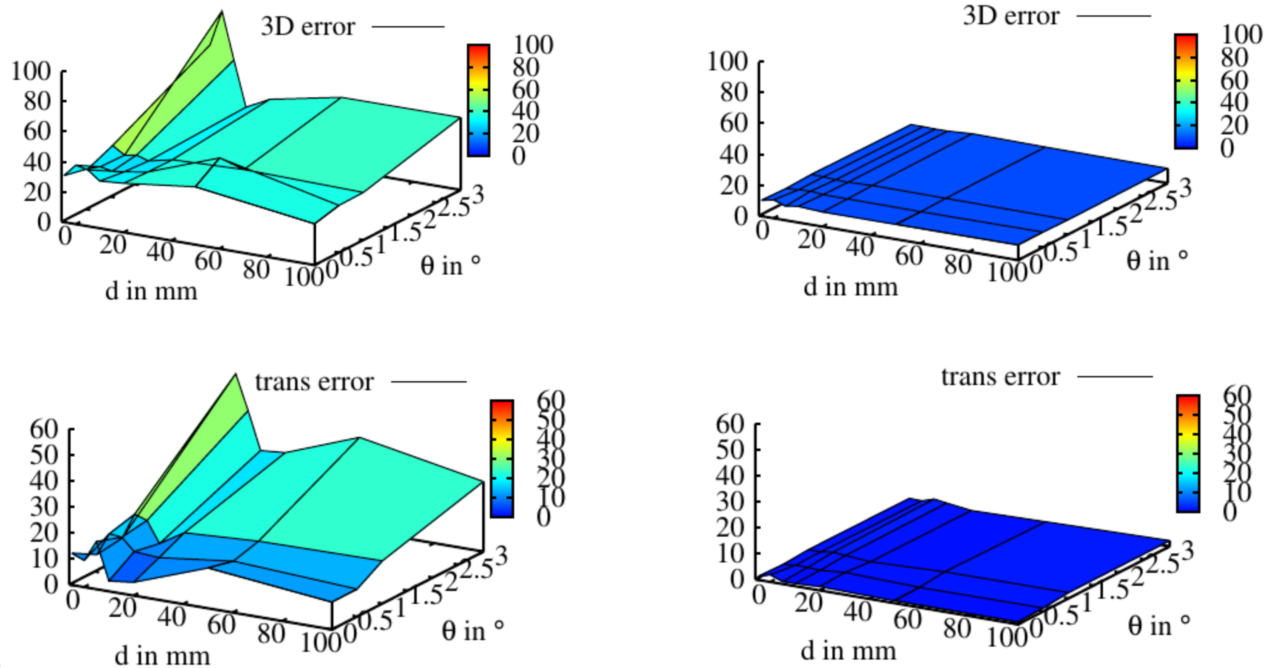


Comparable pipelines for refractive and perspective SfM

Results on Synthetic Images



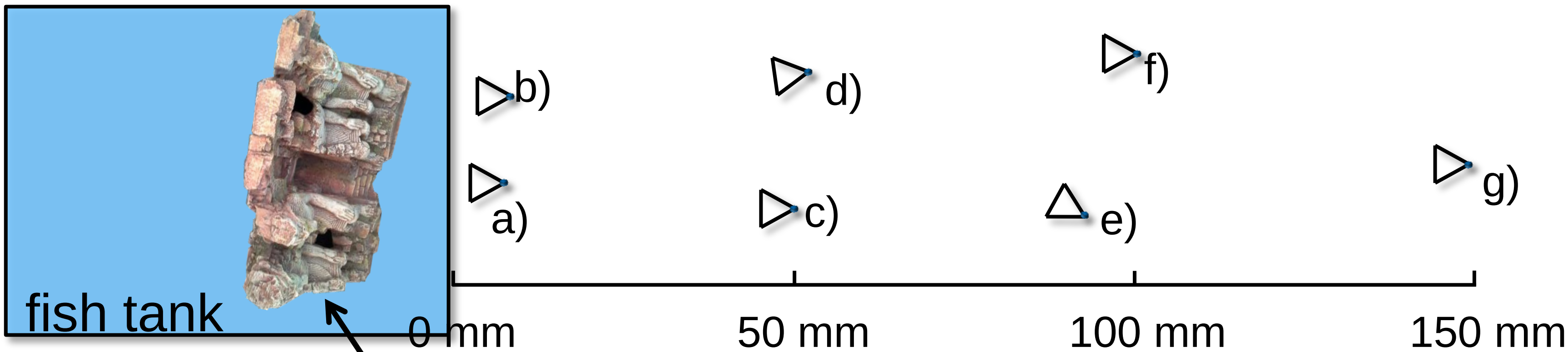
classic perspective | proposed refractive

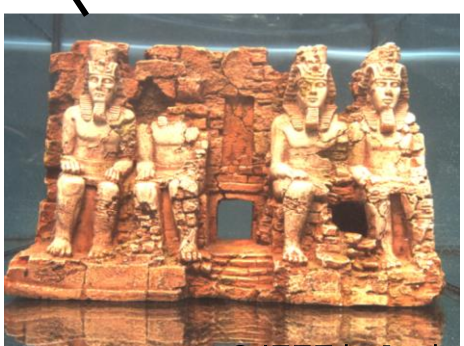


camera-object distance:
4-12 m

No systematic modeling error!

Real Data

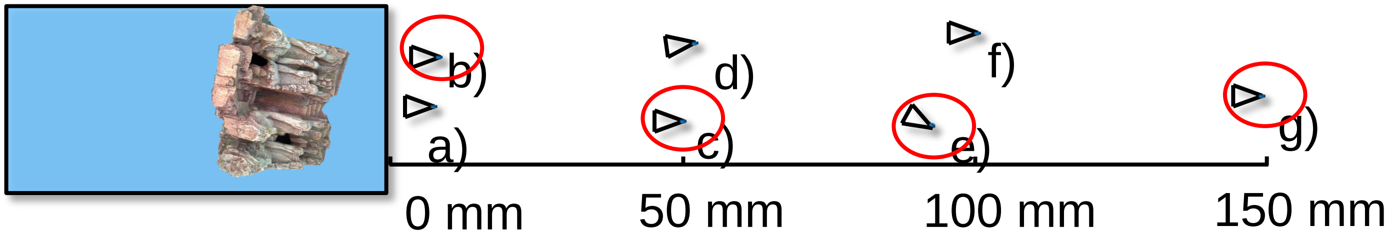


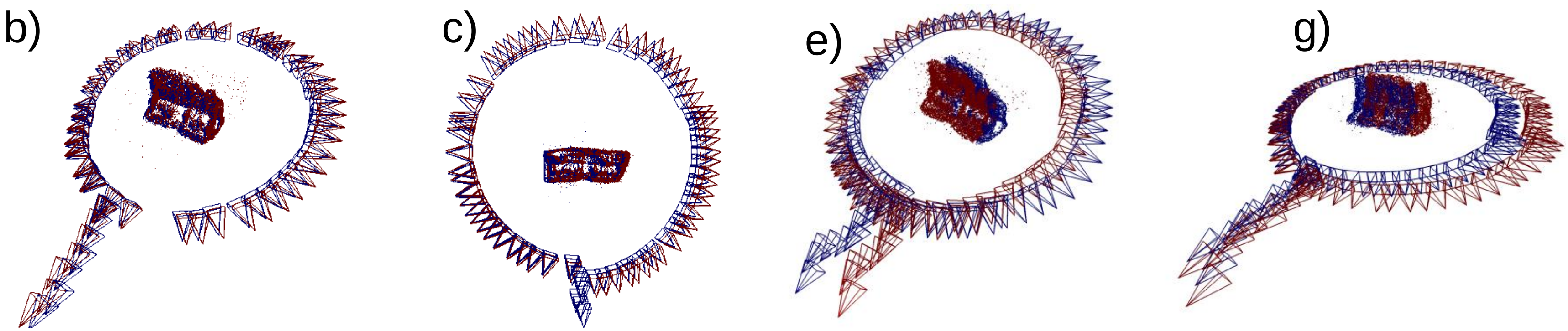


Model: entrance to the Abu Simbel temple (Egypt)

Structure from Motion

Real Data

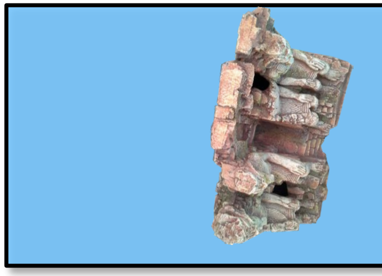


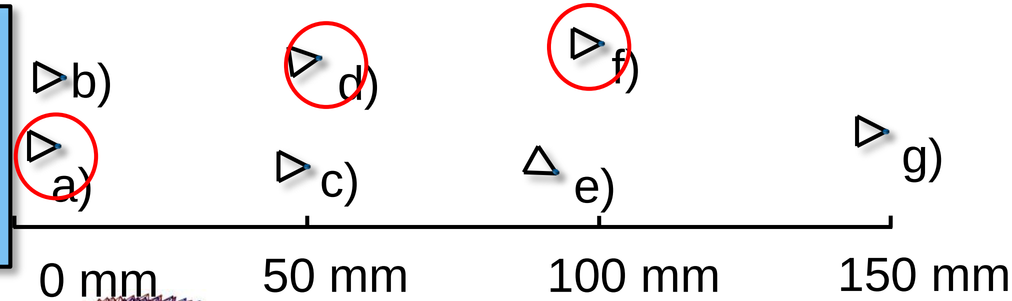


trial	# images	d in mm	θ in °	avg. distance between perspective and refractive pose in mm
b	52	10.60	0.25	24.79
c	67	51.95	0.29	26.44
e	76	76.96	29.29	115.60
g	79	149.39	0.12	79.51

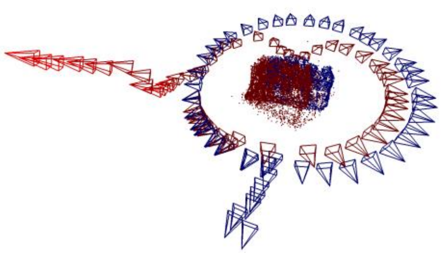
Structure from Motion

Real Data

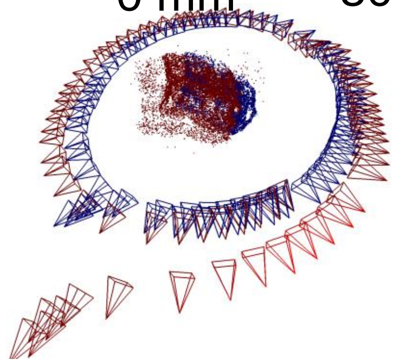




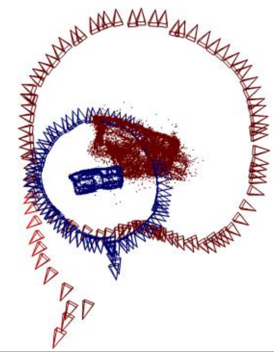
a)



d)



f)



trial	# images	d in mm	θ in °	avg. distance between perspective and refractive pose in mm
a	46	7.88	0.34	350.88
d	65	61.47	7.36	186.57
f	87	95.45	0.12	609.38

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Further Reading

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Wrap Up

- refraction causes systematic modeling error, when ignored in SfM approaches
- by modeling refraction explicitly, this error can be eliminated
- requires SfM approach that does not rely on 3D-2D projections (which are infeasible, especially during non-linear optimization)

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